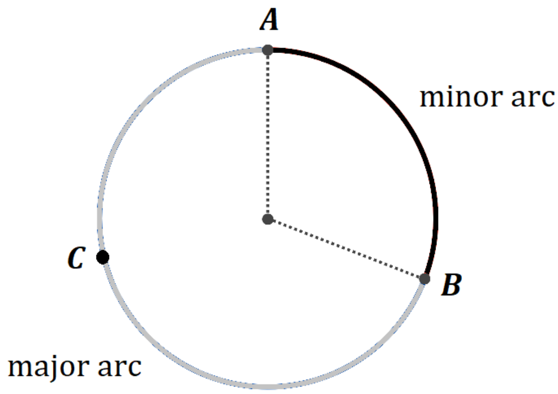
15.2 Arcs and Angles

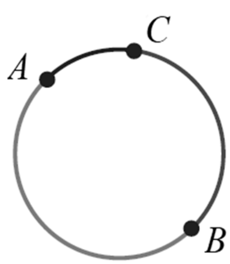


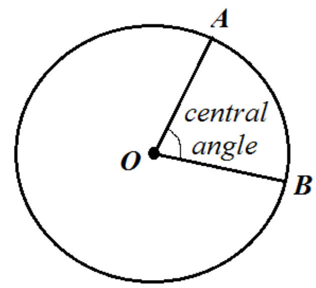
An **arc** is part of the circumference of a circle. A **minor arc** is an arc of less than half of a circle; a **major arc** is more than half of a circle. The symbol for the minor arc between points A and B on a circle is $\overset{\frown}{AB}$. To represent a major arc, a third point on the circle is needed; for example, in the diagram above, $\overset{\frown}{ACB}$ is a major arc.

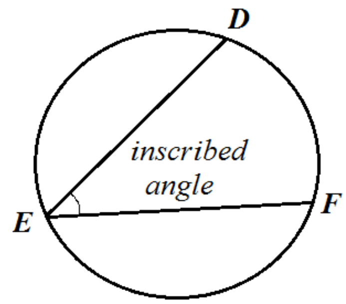
An arc may be measured not only by its length, but also by its measure in degrees.
$\mathrm{m}\,\overset{\frown}{AB}$ represents the measure of arc AB in degrees.

To determine the measure of an arc, multiply the fraction of the circle that the arc represents by 360°. For examples, a semicircle arc is $\frac{1}{2} \cdot 360° = 180°$ and a quarter-circle arc is $\frac{1}{4} \cdot 360° = 90°$.

Adjacent arcs are two arcs of the same circle that share exactly one point in common. The measure of the arc formed by two adjacent arcs is the sum of the measures of the arcs.
Example: $\overset{\frown}{AB}$ and $\overset{\frown}{BC}$ are adjacent arcs. Therefore, $\mathrm{m}\,\overset{\frown}{ABC} = \mathrm{m}\,\overset{\frown}{AB} + \mathrm{m}\,\overset{\frown}{BC}$.







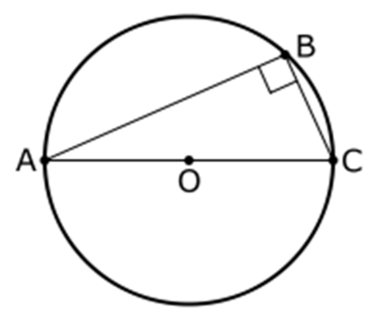
A **central angle** of a circle has its vertex at the center of the circle and sides that are radii.

An **inscribed angle** of a circle has its vertex on the circle and sides that are chords. A chord is a line segment that has both of its endpoints on the circle.

Central Angle Theorem: A central angle is equal in measure to its intercepted arc.
Example: In the diagram above, $\text{m}\angle AOB = \text{m}\,\widehat{AB}$.

Inscribed Angle Theorem: An inscribed angle is equal to half the measure of its intercepted arc.
Example: In the diagram above, $\text{m}\angle DEF = \frac{1}{2} \cdot \text{m}\,\widehat{DF}$.

Semicircle Theorem: An angle inscribed in a semicircle is a right angle.
Example: If A, B and C are points on a circle where $\overline{AC}$ is a diameter of the circle, then $\angle ABC$ is a right angle. Since $\text{m}\,\widehat{AC} = 180°$, it follows that inscribed $\angle ABC$ should measure 90° (half of 180°) and is therefore a right angle.

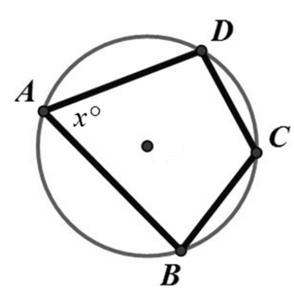


If two arcs of a circle are congruent, then their central angles are congruent. The converse is also true: if two central angles in a circle are congruent, then their intercepted arcs are congruent.

The same could be said for inscribed angles. If two arcs of a circle are congruent, then their inscribed angles are congruent, and vice versa.

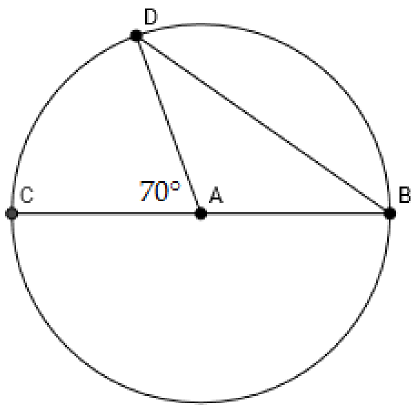
A polygon is called an **inscribed polygon** when all of its vertices lie on a circle.

Opposite Angles Theorem: If a *quadrilateral* is inscribed in a circle, both pairs of opposite angles are supplementary.
Example: Given quadrilateral $ABCD$, let $x = \text{m}\angle A$.
Since $\angle A$ is an inscribed angle, $\text{m}\,\widehat{BCD} = (2x)°$.
Since $\widehat{BCD} + \widehat{BAD} = 360°$, it follows that $\text{m}\,\widehat{BAD} = 360 - 2x$.
$\angle C$ is an inscribed angle, so $\text{m}\angle C = \frac{\text{m}\widehat{BAD}}{2} = \frac{360-2x}{2} = 180 - x$.
So, $\angle A$ and $\angle C$ are supplementary.



MODEL PROBLEM 1: *CENTRAL AND INSCRIBED ANGLES*

Given: $\overline{CB}$ is a diameter of circle A and $m\angle CAD = 70°$.
Find: $m\,\widehat{CD}$, $m\angle CBD$, $m\angle BAD$, $m\,\widehat{BD}$, and $m\angle ADB$.



Solution:

(A) $m\,\widehat{CD} = m\angle CAD = 70°$

(B) $m\angle CBD = \frac{1}{2} m\,\widehat{CD} = 35°$

(C) $m\angle BAD = 180 - 70 = 110°$

(D) $m\,\widehat{BD} = m\angle BAD = 110°$

(E) $m\angle ADB = m\angle CBD = 35°$

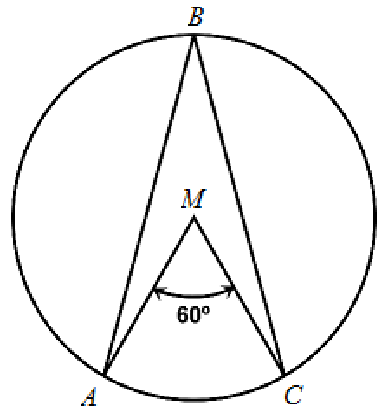
Explanation of steps:

(A) A central angle is equal in measure to its intercepted arc.
(B) An inscribed angle is half the measure of its intercepted arc.
(C) A linear pair adds to 180°.
(D) A central angle is equal in measure to its intercepted arc.
(E) Base angles of an isosceles triangle are equal in measure. *[$\overline{AB}$ and $\overline{AD}$ are radii.]*

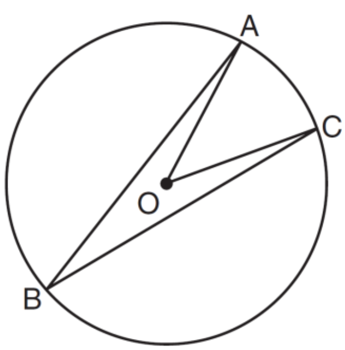
Practice Problems

1. Find the measure of the angle formed by the hands of the clock.

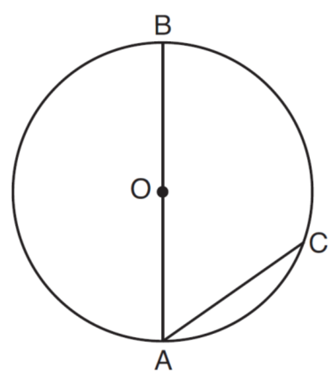
2. In circle M below, $m\angle AMC = 60°$. Find the measure of inscribed $\angle ABC$.



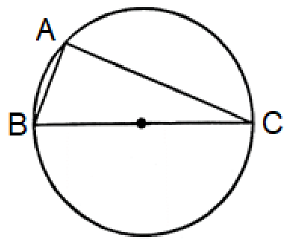
3. In the diagram below of circle O, $m\angle ABC = 24°$. What is the $m\angle AOC$?



4. In the diagram below, $\overline{AB}$ is a diameter of circle O, and chord $\overline{AC}$ is drawn. If $m\angle BAC = 70°$, then what is $m\overparen{AC}$?



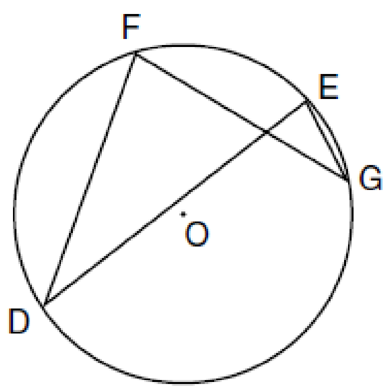
5. $\overline{BC}$ is a diameter of the circle below. If $m\angle B = 55°$, find $m\angle C$.



6. In the diagram of circle O, chords $\overline{DF}$, $\overline{DE}$, $\overline{FG}$, and $\overline{EG}$ are drawn such that

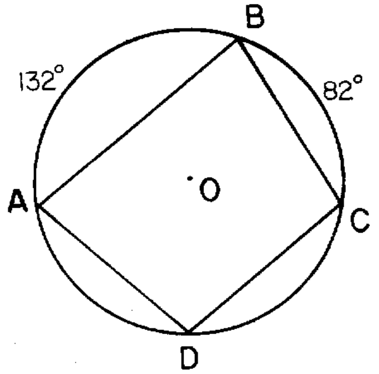
$$m\widehat{DF} : m\widehat{FE} : m\widehat{EG} : m\widehat{GD} = 5 : 2 : 1 : 7$$

Identify two pairs of inscribed angles that are congruent to each other and state their measures in degrees.



Model Problem 2: *Inscribed Quadrilaterals*

Quadrilateral $ABCD$ is inscribed in a circle. Given m $\widehat{AB} = 132°$ and m $\widehat{BC} = 82°$, find m$\angle ABC$.



Solution:

(A) m$\angle ADC = 107°$

(B) m$\angle ABC = 73°$

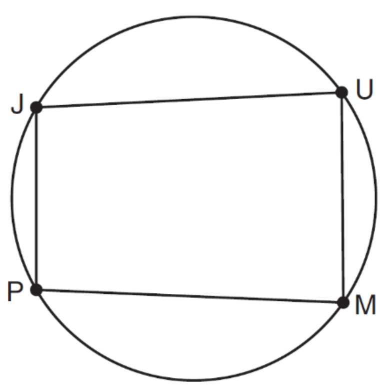
Explanation of steps:

(A) An inscribed angle is half the measure of its intercepted arc.
*[*m$\angle ADC = \frac{1}{2} \cdot$ m $\widehat{ABC} = \frac{1}{2}(132 + 82) = 107°$*]*

(B) In an inscribed quadrilateral, opposite angles are supplementary.
*[*m$\angle ABC = 180 -$ m$\angle ADC = 180 - 107 = 73°$*]*

Practice Problems

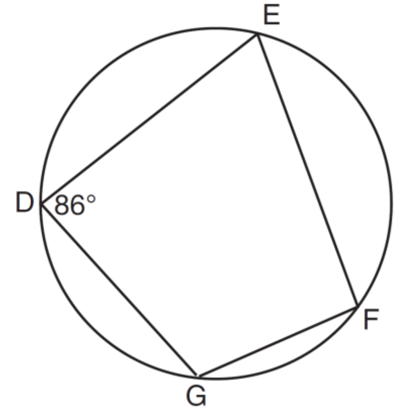
7. In the diagram below, quadrilateral $JUMP$ is inscribed in a circle.



Opposite angles J and M must be
(1) right
(2) complementary
(3) congruent
(4) supplementary

8. Quadrilateral $DEFG$ is inscribed in a circle and m$\angle D = 86°$.
(a) Find m $\widehat{GFE}$.

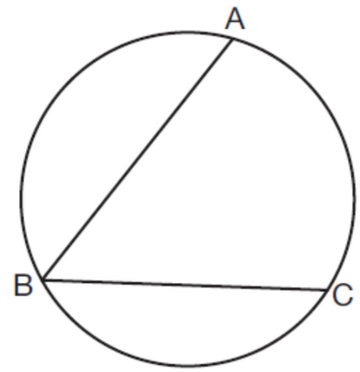
(b) Find m$\angle F$.



Regents Questions

Multiple Choice

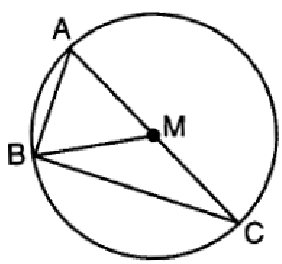
1. In the diagram below, m $\widehat{ABC} = 268°$.



What is the number of degrees in the measure of $\angle ABC$?

(1) 134°
(2) 92°
(3) 68°
(4) 46°

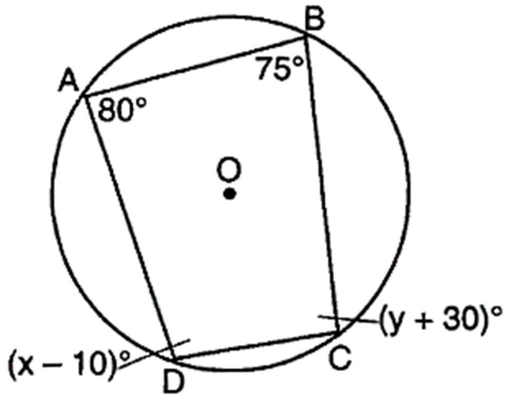
2. In circle M below, diameter $\overline{AC}$, chords $\overline{AB}$ and $\overline{BC}$, and radius $\overline{MB}$ are drawn.



Which statement is *not* true?

(1) $\triangle ABC$ is a right triangle.
(2) $\triangle ABM$ is isosceles.
(3) m $\widehat{BC} = m\angle BMC$
(4) m $\widehat{AB} = \frac{1}{2} m\angle ACB$

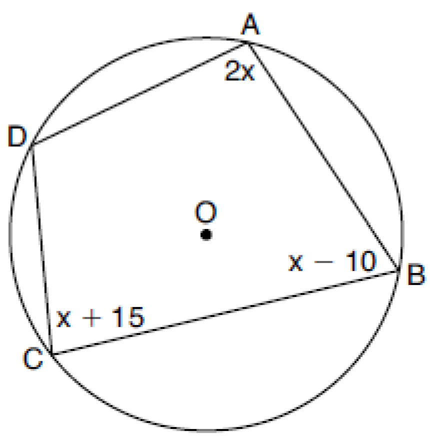
3. Quadrilateral $ABCD$ is inscribed in circle O, as shown below.



If $m\angle A = 80°$, $m\angle B = 75°$, $m\angle C = (y + 30)°$, and $m\angle D = (x - 10)°$, which statement is true?

(1) $x = 85$ and $y = 50$
(2) $x = 90$ and $y = 45$
(3) $x = 110$ and $y = 75$
(4) $x = 115$ and $y = 70$

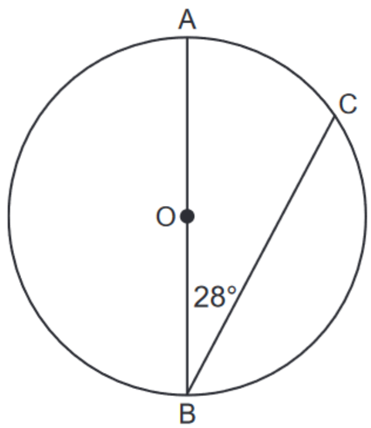
4. In the diagram below, quadrilateral $ABCD$ is inscribed in circle O, $m\angle A = (2x)°$, $m\angle B = (x - 10)°$, and $m\angle C = (x + 15)°$.



What is $m\angle D$?

(1) 55°
(2) 70°
(3) 110°
(4) 135°

5. In the diagram below of Circle O, diameter $\overline{AOB}$ and chord $\overline{CB}$ are drawn, and $m\angle B = 28°$.

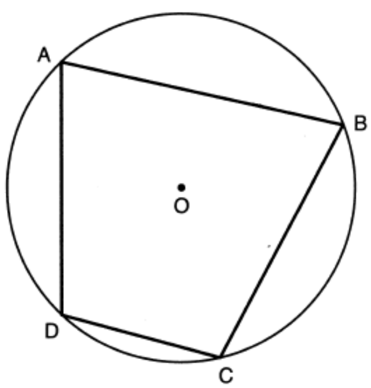


What is m $\widehat{BC}$?

(1) 56°
(3) 152°
(2) 124°
(4) 166°

CONSTRUCTED RESPONSE

6. In the diagram below, quadrilateral $ABCD$ is inscribed in circle O, and m $\widehat{CD}$:m $\widehat{DA}$:m $\widehat{AB}$:m $\widehat{BC}$ $= 2:3:5:5$.



Determine and state $m\angle B$.