Definitions, Postulates and Theorems

This document provides a collection of definitions, postulates and theorems that are most frequently used in geometric proofs.

Section 1.1 Lines, Angles and Shapes

Definition of Congruent Segments:
Congruent segments are segments that have the same length.
$(\overline{AB} \cong \overline{CD}) \leftrightarrow (AB = CD)$

Definition of Congruent Angles:
Congruent angles are angles that have the same measure.
$(\angle A \cong \angle B) \leftrightarrow (\mathrm{m}\angle A = \mathrm{m}\angle B)$

Definition of Midpoint:
A midpoint of a segment is the point that divides the segment into two congruent segments.

Definition of Adjacent Angles:
Adjacent angles are two angles that have a common vertex and common side and don't overlap.

Definitions of Segment Bisector:
A segment bisector intersects a segment at its midpoint and divides the segment into two congruent segments.

Definition of Angle Bisector:
An angle bisector divides an angle into two congruent angles.

Definition of Right Angle:
A right angle is an angle whose measure is 90°.

Definition of Straight Angle:
A straight angle is an angle that measures 180°.

Definition of Complementary Angles:
Two angles are complementary if the sum of their measures is 90°.

Definition of Supplementary Angles:
Two angles are supplementary if the sum of their measures is 180°.

Definition of Linear Pair:
A pair of adjacent angles formed by the intersection of two lines are a linear pair.

Definition of Vertical Angles:
A pair of non-adjacent angles formed by the intersection of two lines are vertical angles.

Definition of Parallel Lines:
Parallel lines are lines on a plane that are everywhere equidistant and never intersect.

Definition of Perpendicular Lines:
Perpendicular lines intersect to form right angles.

Definition of Perpendicular Bisector:
A perpendicular bisector is perpendicular to a segment at its midpoint.

SECTION 1.2 PYTHAGOREAN THEOREM

Pythagorean Theorem:
In a right triangle, if a and b represent the lengths of the legs, and c represents the length of the hypotenuse, then $a^2 + b^2 = c^2$.

SECTION 2.2 PARALLEL AND PERPENDICULAR LINES

Slopes of Parallel Lines:
In a plane, if two distinct lines have the same slope, then they are parallel.

Slopes of Perpendicular Lines:
In a plane, if two lines have slopes that are opposite reciprocals, they are perpendicular.

SECTION 2.3 DISTANCE FORMULA

Distance Formula:
The distance between points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

SECTION 2.4 MIDPOINT FORMULA

Midpoint Formula:
The midpoint of the segment with endpoints of (x_1, y_1) and (x_2, y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

SECTION 3.2 QUADRILATERALS IN THE COORDINATE PLANE

Definition of Quadrilateral:
A quadrilateral is a four-sided polygon.

Definition of Trapezoid:
A trapezoid is a quadrilateral with at least one pair of parallel sides.

Definition of Kite:
A kite is a quadrilateral with two pairs of consecutive congruent sides.

Definition of Parallelogram:
A parallelogram is a quadrilateral with two pairs of parallel sides.

Definition of Rectangle:
A rectangle is an equiangular parallelogram; all four angles are right angles.

Definition of Rhombus:
A rhombus is an equilateral parallelogram; all four sides are congruent.

Definition of Square:
A square is both an equiangular and equilateral parallelogram.

SECTION 6.1 PROPERTIES OF TRANSFORMATIONS

Congruence Transformations:
The image of a polygon or circle after a rigid motion (translation, reflection, or rotation) is congruent to its pre-image.

Similarity Transformations:
The image of a polygon or circle after a dilation is similar to its pre-image.

SECTION 8.1 POSTULATES, THEOREMS AND PROOFS

Right Angles Congruence Postulate:
All right angles are congruent.

Parallel Postulate:
Through an external point, exactly one line may be drawn parallel to a given line.

Perpendicular Postulate:
Through an external point, exactly one line may be drawn perpendicular to a given line.

Partition Postulate:
A whole is equal to the sum of its parts.

Segment Addition Postulate:
Given segment $\overline{ABC}$, then $AB + BC = AC$.

Angle Addition Postulate:
Given two adjacent angles, the measure of the whole angle that contains them both is equal to the sum of the measures of the two angles.

Reflexive Property:
A quantity is equal to itself, or an object is congruent to itself.

Symmetric Property:
The two sides of an equality (or congruence) may be interchanged.

Transitive Property:
If two quantities are equal (or congruent) to the same quantity (or object), then they are equal (or congruent) to each other.

Addition Property of Equality:
If equals are added to equals, their sums are equal.

Subtraction Property of Equality:
If equals are subtracted from equals, their differences are equal.

Multiplication Property of Equality:
If equals are multiplied by equals, their products are equal.

Division Property of Equality:
If equals are divided by equals, their quotients are equal.

Substitution Property:
A quantity may be substituted for its equal in an equation.

Straight Angles Congruence Theorem:
All straight angles are congruent.

Vertical Angles Theorem:
If two angles are vertical angles, then they are congruent.

Linear Pair Theorem:
If two angles form a linear pair, then they are supplementary.

Congruent Complements Theorem:
Complements of the same angle, or of congruent angles, are congruent.

Congruent Supplements Theorem:
Supplements of the same angle, or of congruent angles, are congruent.

Section 8.2 Parallel Lines and Transversals

Definition of Transversal:
A transversal is a line that intersects two other lines in the same plane at different points.

Corresponding Angles Theorem:
If a transversal intersects two parallel lines, corresponding angles are congruent.

Alternate Interior Angles Theorem:
If a transversal intersects two parallel lines, alternate interior angles are congruent.

Alternate Exterior Angles Theorem:
If a transversal intersects two parallel lines, alternate exterior angles are congruent.

Consecutive Interior Angles Theorem:
If a transversal intersects two parallel lines, then consecutive interior angles are supplementary.

Corresponding Angles Converse:
If a transversal intersects two lines to form congruent corresponding angles, then the lines are parallel.

Alternate Interior Angles Converse:
If a transversal intersects two lines to form congruent alternate interior angles, then the lines are parallel.

Alternate Exterior Angles Converse:
If a transversal intersects two lines to form congruent alternate exterior angles, then the lines are parallel.

SECTION 9.1 ANGLES OF TRIANGLES

Definition of Acute Triangle:
An acute triangle has three acute interior angles.

Definition of Right Triangle:
A right triangle has one right angle.

Definition of Obtuse Triangle:
An obtuse triangle has one interior angle that is obtuse.

Triangle Sum Theorem:
The sum of the interior angle measures in a triangle is 180°.

Exterior Angle Theorem:
An exterior angle of a triangle is equal in measure to the sum of the measures of its two remote interior angles.

SECTION 9.2 TRIANGLE INEQUALITY THEOREM

Triangle Inequality Theorem:
The sum of the lengths of the two smallest sides of a triangle is greater than the length of the largest side.

SECTION 9.3 SEGMENTS IN TRIANGLES

Definition of Angle Bisector of a Triangle:
An angle bisector of a triangle is a segment joining a vertex and the opposite side such that it bisects the angle at that vertex.

Definition of Median of a Triangle:
A median of a triangle is a segment joining a vertex and the midpoint of the opposite side.

Definition of Altitude of a Triangle:
An altitude of a triangle is a segment joining a vertex and the line containing the opposite side such that it is perpendicular to that line.

Section 9.4 Isosceles and Equilateral Triangles

Definition of Isosceles Triangle:
An isosceles triangle has at least two congruent sides, called legs.

Definition of Equilateral Triangle:
An equilateral triangle has three congruent sides.

Isosceles Triangle Theorem:
The base angles of an isosceles triangle are congruent.

Isosceles Triangle Converse:
If two angles of a triangle are congruent, then the opposite sides are congruent and the triangle is isosceles.

Section 9.5 Triangle Congruence Methods

Definition of Congruent Triangles:
Two triangles are congruent if their corresponding sides and angles are congruent.

SSS Congruence Theorem:
If three sides of one triangle are congruent to the corresponding sides of another triangle, then the triangles are congruent.

SAS Congruence Theorem:
If two sides and the included angle of one triangle are congruent to the corresponding sides and angle of another triangle, then the triangles are congruent.

ASA Congruence Theorem:
If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.

AAS Congruence Theorem:
If two angles and the non-included side of one triangle are congruent to two angles and the corresponding side of another triangle, then the triangles are congruent.

Section 10.1 Properties of Similar Triangles

Definition of Similar Triangles:
Two triangles are similar if their corresponding angles are congruent and their corresponding sides are in proportion.

Third Angles Theorem:
If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent.

Section 10.2 Triangle Similarity Methods

AA Similarity Theorem:
If two angles of one triangle are equal in measure to two angles of another triangle, then the two triangles are similar.

SSS Similarity Theorem:
If the three sides of one triangle are proportional to the three corresponding sides of another triangle, then the triangles are similar.

SAS Similarity Theorem:
If two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.

Section 10.4 Triangle Angle Bisector Theorem

Triangle Angle Bisector Theorem:
The bisector of an angle of a triangle splits the opposite side into segments that are proportional to the adjacent sides.

Section 10.5 Side Splitter Theorem

Side Splitter Theorem (*also known as the Triangle Proportionality Theorem*):
A line parallel to one side of a triangle divides the other two sides proportionally.

Section 10.6 Triangle Midsegment Theorem

Triangle Midsegment Theorem:
The segment joining midpoints of two sides of a triangle is parallel to the third side and half its length.

Section 11.2 Orthocenter and Centroid

Centroid Median Theorem:
The centroid of a triangle divides each median in a ratio of 2:1, with the larger segment at the vertex. The centroid is the intersection of the three medians.

Section 12.1 Congruent Right Triangles

HL Congruence Theorem:
If the hypotenuse and a leg of one right triangle are congruent to the corresponding hypotenuse and leg of another right triangle, then the triangles are congruent.

Section 12.2 Equidistance Theorems

Angle Bisector Equidistance Theorem:
Any point on an angle bisector is equidistant to the two sides of the angle.

Angle Bisector Equidistance Converse:
If a point in the interior of an angle is equidistant to the two sides of the angle, then it lies on the bisector of the angle.

Perpendicular Bisector Equidistance Theorem:
Any point on the perpendicular bisector of a line segment is equidistant to the endpoints of the line segment.

Perpendicular Bisector Equidistance Converse:
If a point is equidistant from the endpoints of a segment, then it is on the perpendicular bisector of the segment.

SECTION 12.3 GEOMETRIC MEAN THEOREMS

Geometric Mean Theorem (Altitude Rule):
In a right triangle, the altitude drawn from the vertex of the right angle, is the geometric mean between the projections of the legs onto the hypotenuse.

Geometric Mean Theorem (Leg Rules):
In a right triangle, each leg is the geometric mean between the projection of that leg onto the hypotenuse and the hypotenuse.

SECTION 15.2 ARCS AND ANGLES

Central Angle Theorem:
A central angle is equal in measure to its intercepted arc.

Inscribed Angle Theorem:
An inscribed angle is equal to half the measure of its intercepted arc.

Semicircle Theorem:
An angle inscribed in a semicircle is a right angle.

Opposite Angles Theorem:
If a quadrilateral is inscribed in a circle, both pairs of opposite angles are supplementary.

SECTION 15.3 CHORDS

Chords of a Circle Theorems:
If two arcs are congruent, their chords are congruent, and vice versa.
If two central angles are congruent, their chords are congruent, and vice versa.
If two inscribed angles are congruent, their chords are congruent, and vice versa.

Parallel Chords Theorem:
If two chords are parallel, they intercept congruent arcs between the chords.

Perpendicular Chord Bisector Theorem:
If a diameter (or radius) is perpendicular to a chord, it bisects the chord and its arc.

Equidistant Chords Theorem:
If two chords in a circle are equidistant from the center, then they are congruent.

Angle of Intersecting Chords Theorem:
Two intersecting chords form a pair of vertical angles that are each equal in measure to the average of their intercepted arcs.

Intersecting Chords Theorem:
When two chords intersect in a circle, the product of the segments of one chord equals the product of the segments of the other chord.

SECTION 15.4 TANGENTS

Tangent-Radius Theorem:
The tangent of a circle is perpendicular to a radius at the point of tangency.

Tangent-Chord Theorem:
The angle formed by a tangent and a chord is half the measure of the intercepted arc.

Two Tangent Theorem:
The two tangent segments drawn to a circle from the same external point are congruent.

Circumscribed Angle Theorem:
A circumscribed angle is equal to 180° minus the measure of the minor arc it intercepts.

SECTION 15.5 SECANTS

Intersecting Secant Angles Theorem:
If two secants (or two tangents, or a secant and a tangent) intersect at an external point, then the measure of the angle formed is half the difference of the intercepted arcs.

Intersecting Secants Theorem:
If two secants intersect at an external point, the product of the lengths of one secant and its external part equals the product of the lengths of the other secant and its external part.

Corollary to Intersecting Secants Theorem:
When a secant and tangent intersect at an external point, the product of the lengths of the secant and its external part equals the square of the length of the tangent.