Answer Key

Geometry Regents
Course Workbook

with Exam Questions

2019-20 Edition

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Notation

A code next to each Regents Question number states from which Geometry Common Core Regents exam the question came. For example, AUG '18 [25] means the question appeared on the August 2018 as question 25.

2. PERIMETER AND AREA

2.1 Perimeter and Circumference

PRACTICE PROBLEMS

1.
$x+(x+2)+(x-3)$
Perimeter is $3x-1$

2.
$$4x+(x+3)+3x-1=34$$
$$8x+2=34$$
$$8x=32$$
$$x=4$$
$4x=16,\ x+3=7,\ 3x-1=11$

3.
Length of rectangle = diameter = 8 in.
Perimeter = $\frac{1}{2}2\pi r+l+2w$
$=\frac{1}{2}8\pi+8+14=4\pi+22\approx 34.6$ in.

4.
Width of rectangle = radius = 2 cm.
Perimeter = $\frac{1}{2}2\pi r+2l+w$
$=\frac{1}{2}4\pi+8+2=2\pi+10\approx 16.3$ cm.

5. $\frac{1}{2}15\pi+60+15=7.5\pi+75\approx 98.6$ ft.

6. $\frac{1}{2}\cdot 4\pi+4\cdot 4=2\pi+16\approx 22.3$ cm.

7. $\frac{1}{2}12\pi+8+14=6\pi+22\ m$

8. $\frac{1}{2}6\pi+4+10+6=3\pi+20$

9.
All sides of the polygon are equal to 3.5.
Perimeter $= 3.5\times 7 = 24.5$ inches

10.
Each side $s = 2r = 10$
Perimeter = $4\left(\frac{1}{4}2\pi r\right)+4s$
$= 10\pi + 40$

11.
Length of arc $SBT = \frac{1}{4}2\pi r=\frac{1}{4}12\pi=3\pi$.
Let $w=RC$, so $AR=8-w$.
$CT=r-RC=6-w$
$AS=r-AR=6-(8-w)=w-2$
Since $ABCR$ is a rectangle, its diagonals are congruent.
Diagonal RB is a radius, so $AC=RB=6$.
Perimeter =
$\overset{\frown}{SBT}+CT+AS+AC=$
$3\pi+(6-w)+(w-2)+6=$
$3\pi+10$

2.2 Area

Practice Problems

1. $\frac{1}{2}\pi r^2 + s^2 = \frac{1}{2}36\pi + 12^2 = 18\pi + 144$	2. $\pi r^2 + lw = 5^2\pi + (20)(10) = 25\pi + 200$
3. $s^2 + \pi r^2 = (2x)^2 + \pi x^2$ $= 4x^2 + \pi x^2$	4. $s^2 - \pi r^2 = 8^2 - 4^2\pi = 64 - 16\pi$
5. $s = \sqrt{36} = 6 \qquad r = \frac{1}{2}s = 3$ $A = \pi r^2 = 9\pi$	6. $A = s^2 - \frac{1}{2}\pi r^2 = 6^2 - \frac{1}{2}9\pi = 36 - 4.5\pi$
7. $lw - 2\pi r^2 = (20)(10) - 2(5^2)\pi = 200 - 50\pi$	8. $\frac{1}{4}\pi r^2 - \frac{1}{2}bh = \frac{1}{4}(4^2)\pi - \frac{1}{2}(4)(4) = 4\pi - 8$
9. $lw - \frac{1}{2}bh = (12)(4) - \frac{1}{2}(9)(4) = 30$	10. $s^2 - \pi r^2 = 6^2 - 3^2\pi = 36 - 9\pi$
11. $lw - \pi r^2 = (36)(15) - 4^2\pi = 540 - 16\pi \approx 490$ sq. ft.	$490 \times 1.95 = \$955.50$

Regents Questions

1) JAN '17 [8] Ans: 1
2) AUG '17 [20] Ans: 1
3) JAN '19 [18] Ans: 1
4) JAN '19 [31]
Area of outer circle – Area of inner circle $= \pi 30^2 - \pi 20^2 = 500\pi$
Area of each rectangle
$= 90 \times 10 = 900$
Total area = $500\pi + 2(900) \approx$ 3,371 sq. ft

3. LINES, ANGLES AND PROOFS

3.1 Parallel Lines and Transversals

PRACTICE PROBLEMS

1. (a) linear pair; supplementary
   (b) vertical angles; congruent
   (c) corresponding angles; congruent
   (d) alternate interior angles; congruent
   (e) alternate exterior angles; congruent

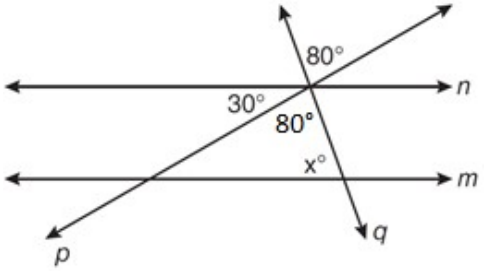
2. (2) Alternate interior angles are congruent.

3. (3) $\angle 3$ and $\angle 6$ are supplementary

4. (4) $d \parallel e$

5. $m\angle EYD = 180 - 123 = 57°$ (linear pair)
   $m\angle AXY = m\angle EYD = 57°$ (alternate interior)

6. Vertical angles are congruent, therefore:



$x = 180 - (30 + 80) = 70$

7. $a = 180 - (57 + 64) = 59°$
   $b = 64°$
   $c = 57°$
   $d = 180 - 64 = 116°$

8.
$$15x - 5 = 180 - 125$$
$$15x - 5 = 55$$
$$15x = 60$$
$$x = 4$$

$$7y + 27 = 125$$
$$7y = 98$$
$$y = 14$$

REGENTS QUESTIONS

1) JUN '15 [17] Ans: 1
2) AUG '16 [1] Ans: 2

3.2 Deductive Reasoning

Practice Problems

1. (4) vertical angles	2. (1) $\overline{AC} \cong \overline{DB}$
3. (2) $AB = CD$	4. (2) reflexive property and subtraction postulate

5.

Statements	*Reasons*
$\angle 1$ and $\angle 2$ are complementary	Given
$\angle 2$ and $\angle 3$ are complementary	Given
$\angle 1 \cong \angle 3$	Complements of the same angle are congruent
$m\angle 1 = m\angle 3$	Definition of congruent angles (Congruent angles are equal in measure)

6.

Statements	*Reasons*
$\overline{PCEG}, \overline{PC} \cong \overline{GE}$	Given
$\overline{CE} \cong \overline{CE}$	Reflexive Postulate
$PC = GE, \quad CE = CE$	Definition of congruent segments
$PC + CE = GE + CE$	Addition Postulate
$PE = PC + CE,$ $= GE + CE$	Partition Postulate
$PE = GC$	Substitution Postulate
$\overline{PE} \cong \overline{GC}$	Definition of congruent segments

7.

Statements	*Reasons*
$\angle 1$ and $\angle 3$ are supplementary	Given
$\angle 1$ and $\angle 2$ are a linear pair	Definition of linear pair
$\angle 1$ and $\angle 2$ are supplementary	Linear pairs are supplementary
$\angle 2 \cong \angle 3$	Supplements of the same angle are congruent
$\angle 2$ and $\angle 3$ are alternate exterior angles	Definition of alternate exterior angles
$m \parallel n$	If two lines cut by a transversal form congruent alternate exterior angles, then the lines are parallel

Regents Questions

1) JAN '16 [6] Ans: 1

2) JAN '16 [33]
(2) Parallel Postulate;
(3) Alternate interior angles formed by parallel lines and a transversal are congruent;
(4) Consecutive adjacent angles on a straight line add to 180°;
(5) Substitution

4. TRIANGLES

4.1 Triangle Inequality Theorem

Practice Problems

1. (2)	2. (4)
3. (1)	4. The sum of any two sides of a triangle must be greater than the third side. $7 + 8 < 16$.
5. (2)	6. $4x + 3x - 1 + x + 3 = 34$ $8x + 2 = 34$ $8x = 32$ $x = 4$ 16, 11, and 7. Yes, because $16 < 11 + 7$.
7. $\angle B, \angle A, \angle C$	8. $\overline{EF}$, $\overline{DE}$, $\overline{DF}$

Regents Questions

1) JAN '19 [19] Ans: 3

4.2 Angles of Triangles

Practice Problems

1. $180 - (60 + 40) = 80$

2. $50 + 70 = 120°$

3. $\text{m}\angle ABC = 180 - 125 = 55°$
   $\text{m}\angle ACR = 60 + 55 = 115°$

4. $60 + 60 = 120°$

5.
$$6x + 5 = 3x + 65$$
$$3x = 60$$
$$x = 20$$

6.
$$5x + x + 12 + x = 180$$
$$7x + 12 = 180$$
$$7x = 168$$
$$x = 24$$
Angles are 24°, 36°, and 120°, so the triangle is obtuse.

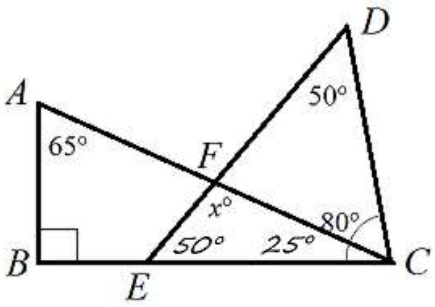
7.
$\text{m}\angle J = 180 - (90 + 48) = 42°$
$\text{m}\angle JMS = 180 - 59 = 121°$
$\text{m}\angle JSM = 180 - (42 + 121) = 17°$

8.
$\text{m}\angle EHI = 20°$ (vertical angles)
$\text{m}\angle EIH = 60°$ (vertical angles)
$\text{m}\angle HEI = 180 - (20 + 60) = 100°$

9.
$\text{m}\angle RTQ = 63°$ (alternate interior)
$\text{m}\angle 2 = 180 - (90 + 63) = 27°$

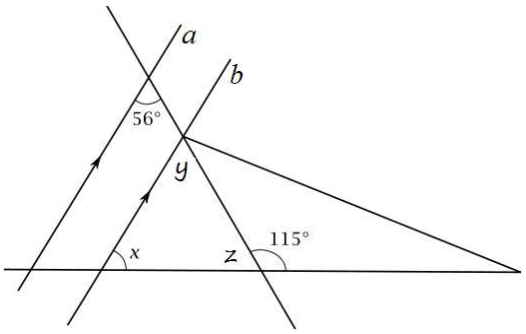
10.
$\text{m}\angle A = 180 - (90 + 52) = 38°$
$\text{m}\angle EBC = 90 + 38 = 128°$
$\text{m}\angle EBD = \frac{1}{2}\text{m}\angle EBC = 64°$
$\text{m}\angle ABD = 52 + 64 = 116°$
$\text{m}\angle D = 180 - (38 + 116) = 26°$

11.



$\text{m}\angle ACB = 180 - (90 + 65) = 25°$
$\text{m}\angle DEC = 180 - (80 + 50) = 50°$
$x = 180 - (50 + 25) = 105°$

12.



$y = 56°$ (corresponding angles)
$z = 65°$ (linear pair)
$x = 180 - (56 + 65) = 59°$

13. $\text{m}\angle ABD = 180 - (93 + 43) = 44$
$$x + 19 + 2x + 6 + 3x + 5 = 180$$
$$6x + 30 = 180$$
$$x = 25$$
$\text{m}\angle BDC = x + 19 = 44$
Yes, because alternate interior angles $\angle ABD$ and $\angle BDC$ are congruent, $\overline{AB} \parallel \overline{DC}$.

Regents Questions

1) AUG '16 [4] Ans: 2
2) JUN '17 [17] Ans: 4
3) JAN '18 [9] Ans: 3
4) JAN '18 [18] Ans: 2
5) JUN '18 [2] Ans: 3
6) AUG '18 [1] Ans: 4
7) JAN '19 [16] Ans: 4

8) FALL '14 [10]
The sum of the measures of the angles of a triangle is 180°, so $m\angle ABC + m\angle BCA + m\angle CAB = 180°$. Each interior angle of the triangle and its exterior angle form a linear pair. Linear pairs are supplementary, so $m\angle ABC + m\angle FBC = 180°$, $m\angle BCA + m\angle DCA = 180°$, and $m\angle CAB + m\angle EAB = 180°$. By addition, the sum of these linear pairs is 540°. When the angle measures of the triangle are subtracted from this sum, the result is 360°, the sum of the exterior angles of the triangle.

4.3 Medians, Altitudes and Bisectors

PRACTICE PROBLEMS

1. (a) $\overline{BE}$ (b) $\overline{AD}$ (c) $\overline{CF}$	2. (a) circumcenter (b) centroid (c) orthocenter (d) incenter
3. (3) an obtuse triangle	4. (4) circumcenter and orthocenter
5. (1) centroid	6. (a) 48 (b) 16 (c) 24
7. (4) $\angle DBG \cong \angle EBG$	8. $\text{m}\angle REC = \frac{1}{2}(84) = 42$ $\text{m}\angle BRC = 180 - (42 + 28) = 110°$

9. (a)

$$2x + 3 = 7x - 47$$
$$50 = 5x$$
$$10 = x$$
$$DH = 2(10) + 3 = 23$$
$$DG = 2 \cdot DH = 46$$

(b)

$$y^2 + 9 = 90$$
$$y^2 = 81$$
$$y = 9$$
$$\text{m}\angle EFH = 12y = 108°$$

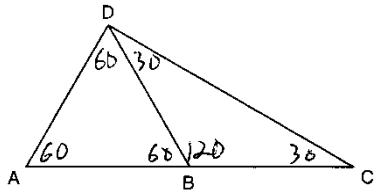
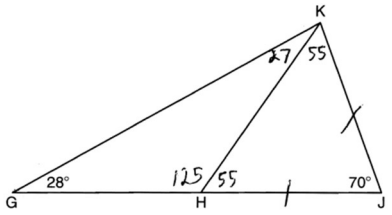
4.4 Isosceles and Equilateral Triangles

Practice Problems

1. $\frac{180-120}{2}=30°$	2. $\frac{180-46}{2}=67°$
3. (a) $\overline{CE}$ (b) $\overline{AF}$ (c) $\overline{BD}$	4. (3) an obtuse angle
5. $m\angle A=\frac{180-80}{2}=50°$ $m\angle BCD=80+50=130°$	6. $m\angle LMO=55°$ $m\angle NMO=180-55=125°$ $m\angle N=180-(125+28)=27°$

7.

(a) $60°$	(equilateral triangle)
(b) $\frac{180-5}{2}=65°$	(base angle of isosceles triangle)
(c) $180-(60+65)=55°$	(parts of a straight angle)
(d) $65+50=115°$	(exterior angle)
(e) $60+60=120°$	(exterior angle)
(f) $60°$	(vertical angles with $\angle BAC$)

8. $m\angle QRP=\frac{180-54}{2}=63°$ $m\angle QRS=180-63=117°$ $x=\frac{180-117}{2}=31.5°$	9. $m\angle EFG=90-60=30°$ $x=180-(90+30)=60°$
10. 30 	11. No, $m\angle KGH \neq m\angle GKH$ 

12.

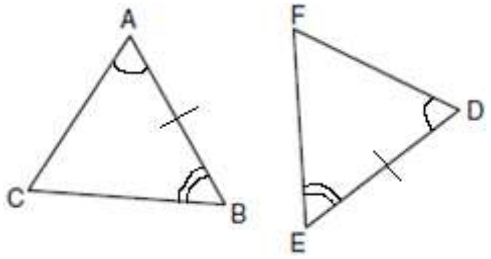
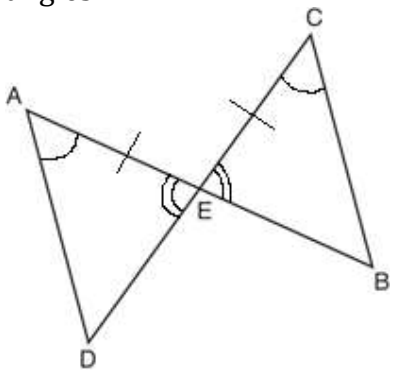
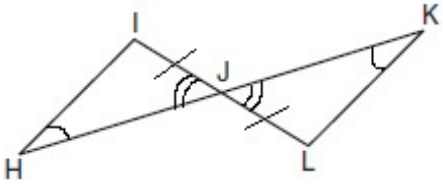
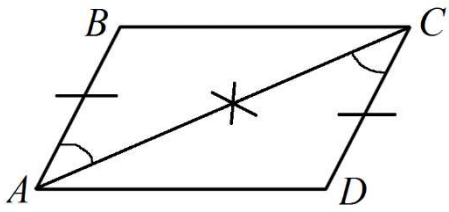
Statements	*Reasons*
$\triangle JKL$, side $\overline{JL}$ extended through L to M, side $\overline{KL}$ extended through L to N, $\text{m}\angle K = 70°$, $\text{m}\angle MLN = 55°$	Given
$\angle JLK$ and $\angle MLN$ are vertical angles	Definition of vertical angles
$\text{m}\angle JLK = \text{m}\angle MLN$	Vertical angles are equal in measure
$\text{m}\angle JLK = 55°$	Substitution
$\text{m}\angle J + \text{m}\angle K + \text{m}\angle JLK = 180°$	Sum of the angles of a triangle is 180°
$\text{m}\angle J + 70° + 55° = 180°$ $\text{m}\angle J = 55°$	Substitution, and solving
$\text{m}\angle JLK = \text{m}\angle J$	Transitive property
$\triangle JKL$ is isosceles	If two angles of a triangle are equal in measure, then the triangle is isosceles

Regents Questions

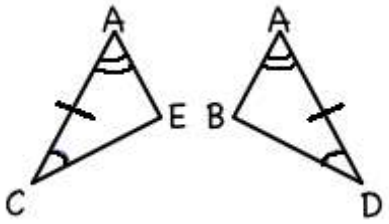
1) JUN '16 [19] Ans: 2
2) AUG '16 [11] Ans: 4
3) AUG '17 [11] Ans: 4
4) AUG '18 [22] Ans: 4
5) FALL '14 [5]
In an isosceles triangle, the bisector of the vertex angle is also a median. Therefore, $MO = \frac{1}{2}OP$, so $MO = 8$.
6) FALL '14 [24]
$\triangle XYZ$, $\overline{XY} \cong \overline{ZY}$ and $\overline{YW}$ bisects $\angle XYZ$ (Given).
$\triangle XYZ$ is isosceles (Definition of isosceles triangle).
$\overline{YW}$ is an altitude of $\triangle XYZ$ (The angle bisector of the vertex of an isosceles triangle is also the altitude of that triangle).
$\overline{YW} \perp \overline{XZ}$ (Definition of altitude).
$\angle YWZ$ is a right angle (Definition of perpendicular lines).
7) JUN '15 [32]
Since linear pairs are supplementary, $m\angle GIH = 65°$.
Since $\overline{GH} \cong \overline{IH}$, $m\angle IGH = 65°$ and $m\angle GHI = 180 - (65 + 65) = 50°$.
Since $\angle EGB \cong \angle GHI$, the corresponding angles formed by the transversal and lines are congruent and $\overline{AB} \| \overline{CD}$.
8) JAN '17 [30]
$m\angle DAC = m\angle ECA = 25°$
$m\angle AXC = 180 - 2(25) = 130°$

4.5 Congruent Triangles

Practice Problems

1. (2) by SSS	2. (1) by SAS
3. ASA  	4. ASA $\angle AED$ and $\angle CEB$ are a pair of vertical angles. 
5. AAS $\overline{IJ} \cong \overline{LJ}$ by definition of bisector $\angle IJH \cong \angle LJK$ by vertical angles  	6. SAS $\overline{CA} \cong \overline{AC}$ by Reflexive Property 
7. AAS $\angle BAC \cong \angle DAC$ by def. of $\angle$ bisector $\overline{AC} \cong \overline{AC}$ by Reflexive Property	8. AAS $\angle BAC \cong \angle DCA$ by alternate interior angles $\overline{AC} \cong \overline{CA}$ by Reflexive Property
9. (1) $\angle A \cong \angle L$	10. $\overline{AG} \cong \overline{OL}$
11. (2) $\angle A \cong \angle X$	12. (3) $\overline{JL} \cong \overline{MO}$
13. 3. Perpendicular line segments form right angles 6. If two parallel lines are cut by a transversal, the alternate interior angles are congruent 8. AAS 9. CPCTC	

14.



Statements		*Reasons*
$\angle C \cong \angle D$	(A)	Given
$\overline{AC} \cong \overline{AD}$	(S)	Given
$\angle A \cong \angle A$	(A)	Reflexive Property
$\triangle ACE \cong \triangle ADB$		ASA
$\overline{CE} \cong \overline{DB}$		CPCTC

15.

$\overline{BE}$ and $\overline{AD}$ intersect at point C, $\overline{BC} \cong \overline{EC}$, $\overline{AC} \cong \overline{DC}$, $\overline{AB}$ and $\overline{DE}$ are drawn (Given)

$\angle BCA \cong \angle ECD$ (vertical angles)

$\triangle ABC \cong \triangle DEC$ (SAS)

16.

$\overline{AD}$ bisects $\overline{BC}$ at E, $\overline{AB} \perp \overline{BC}$, $\overline{DC} \perp \overline{BC}$ (Given)

$\angle B$ and $\angle C$ are right angles (perpendicular lines form right angles)

$\overline{BE} \cong \overline{CE}$ (definition of segment bisector)

$\angle B \cong \angle C$ (all right angles are congruent)

$\angle AEB \cong \angle DEC$ (vertical angles are congruent)

$\triangle ABE \cong \triangle DCE$ (ASA)

$\overline{AB} \cong \overline{DC}$ (CPCTC)

17.

$\triangle ABC$, $\overline{BD}$ bisects $\angle ABC$, $\overline{BD} \perp \overline{AC}$ (Given)

$\angle CBD \cong \angle ABD$ (definition of angle bisector)

$\overline{BD} \cong \overline{BD}$ (Reflexive property)

$\angle CDB$ and $\angle ADB$ are right angles (definition of perpendicular)

$\angle CDB \cong \angle ADB$ (all right angles are congruent)

$\triangle CDB \cong \triangle ADB$ (ASA)

$\overline{AB} \cong \overline{CB}$ (CPCTC)

Regents Questions

1) AUG '16 [22] Ans: 3
2) JAN '17 [3] Ans: 1
3) JUN '17 [9] Ans: 2
4) AUG '18 [10] Ans: 4
5) JUN '17 [33]
$\overline{RS}$ and $\overline{TV}$ bisect each other at point X; $\overline{TR}$ and $\overline{SV}$ are drawn (Given); $\overline{TX} \cong \overline{XV}$ and $\overline{RX} \cong \overline{XS}$ (segment bisectors create two congruent segments); $\angle TXR \cong \angle VXS$ (vertical angles); $\triangle TXR \cong \triangle VXS$ (SAS); $\angle T \cong \angle V$ (CPCTC); $\overline{TR} \parallel \overline{SV}$ (congruent alternate interior angles).
6) AUG '17 [35]
(Givens omitted)
$\overline{AD} \cong \overline{BC}$ (congruent legs of isosceles trapezoid).
$\angle DEA$ and $\angle CEB$ are right angles (perpendicular lines form right angles).
$\angle DEA \cong \angle CEB$ (all right angles are congruent).
$\angle CDA \cong \angle DCB$ (base angles of an isosceles trapezoid are congruent).
$\angle CDA - \angle CDE \cong \angle DCB - \angle DCE$ (subtraction postulate).
$\angle ADE \cong \angle BCE$ (substitution).
$\triangle ADE \cong \triangle BCE$ (AAS).
$\overline{EA} \cong \overline{EB}$ (CPCTC).
$\triangle AEB$ is an isosceles triangle (two congruent sides).
7) AUG '18 [32]
(2) Reflexive; (4) $\angle BDA \cong \angle BDC$; (6) CPCTC; (7) If points B and D are equidistant from the endpoints of $\overline{AC}$, then B and D are on the perpendicular bisector of $\overline{AC}$.

4.6 Similar Triangles

Practice Problems

1. $\frac{25}{10}=\frac{AC}{6}$ $10\cdot AC=150$ $AC=15$	2. $\frac{50}{XY}=\frac{40}{20}$ $40\cdot XY=1000$ $XY=25$
3. $m\angle P = 25°$ and $m\angle R = 45°$, so $m\angle Q = 180 - (25 + 45) = 110°$	
4. $\triangle QRS \sim \triangle UTS$ $\angle Q \cong \angle U$ and $\angle R \cong \angle T$ by alternate interior angles formed by parallel lines. The triangles are similar by AA~. (Also, $\angle RSQ \cong \angle TSU$ by vertical angles)	5. $\triangle AEB \sim \triangle CED$ $\angle A \cong \angle DCE$ and $\angle B \cong \angle CDE$ by corresponding angles formed by parallel lines. The triangles are similar by AA~. (Also, $\angle E \cong \angle E$ by Reflexive Property.)
6. $\frac{30}{21}=\frac{ML}{7}$ $ML=10$	7. $\frac{9}{36}=\frac{4}{x}$ $9x=144$ $x=16$
8. $5x + 8x + 11x = 60$ $24x = 60$ $x = 2.5$ $5x = 5(2.5) = 12.5$	9. $\frac{15}{18}=\frac{5}{6}$
10. $m\angle B = 180 - (50 + 30) = 100$ So, $m\angle X = 100$	11. (3) $\angle ACB \cong \angle DFE$ (the included angles for SAS similarity)
12. SAS	13. AA
14. (4)	
15. $\angle ACB \cong \angle AED$ (Given) $\angle A \cong \angle A$ (Reflexive property) $\triangle ABC \sim \triangle ADE$ (AA~)	16. $\angle B$ and $\angle E$ are right angles (definition of perpendicular lines) $\angle B \cong \angle E$ (all right angles are congruent) $\angle BFD$ and $\angle DFE$ are supplementary and $\angle ECA$ and $\angle ACB$ are supplementary (linear pairs) $\angle DFE \cong \angle ACB$ (angles supplementary to congruent angles are congruent) $\triangle ABC \sim \triangle DEF$ (AA~)

Regents Questions

1) JUN '15 [15] Ans: 3
2) JUN '15 [21] Ans: 4
3) AUG '15 [19] Ans: 2
4) JAN '16 [13] Ans: 1
5) JAN '16 [20] Ans: 4
6) JAN '16 [24] Ans: 3
7) JUN '16 [5] Ans: 3
8) JUN '16 [17] Ans: 1
9) AUG '16 [12] Ans: 3
10) AUG '17 [5] Ans: 4
11) AUG '17 [9] Ans: 4
12) JAN '18 [13] Ans: 3
13) JAN '18 [17] Ans: 4
14) JUN '18 [4] Ans: 3
15) JUN '18 [9] Ans: 4
16) JAN '19 [8] Ans: 1
17) AUG '15 [29]
$\frac{6}{14}=\frac{9}{21}$; Yes (SAS~)
18) JAN '17 [29]
$\overline{GI}$ is parallel to $\overline{NT}$, and $\overline{IN}$ intersects $\overline{GT}$ at A (Given); $\angle I \cong \angle N$, $\angle G \cong \angle T$ (parallel lines cut by a transversal form congruent alternate interior angles); $\triangle GIA \sim \triangle TNA$ (AA~).
19) JUN '18 [30]
Yes, both triangles are 5-12-13 triangles by the Pythagorean Theorem, so they are congruent by SSS. All congruent triangles are also similar.
20) AUG '18 [29]
Triangles are similar by AA. The triangles share the same angle at the stake, so these angles are congruent; the angles at the bases of the two poles are corresponding angles formed by parallel lines, so they are congruent. (The angles formed by the poles with the support wire are also congruent since they are corresponding angles formed by parallel lines.)

4.7 Angle Bisector Theorem

Practice Problems

1. $\frac{x}{4}=\frac{9}{6}$ $6x=36$ $x=6$	2. $\frac{2}{x-2}=\frac{5}{9}$ $5(x-2)=18$ $5x-10=18$ $5x=28$ $x=5.6$

4.8 Triangle Midsegment Theorem

Practice Problems

1. $x = \frac{1}{2}(24) = 12$	2. $5x = 2(2x + 2)$ $5x = 4x + 4$ $x = 4$ $DE = 2(4) + 2 = 10$ $AC = 5(4) = 20$
3. $AB = 36 \div 3 = 12$ $EF = \frac{1}{2}AB = \frac{1}{2} \cdot 12 = 6$ cm (triangle midsegment theorem)	4. $BE = \frac{1}{2}BC = 6$ (def. of midpoint) $EF = \frac{1}{2}AB = 10$ (midsegment theorem) $AF = \frac{1}{2}AC = 8$ (def. of midpoint) Perimeter = $AB + BE + EF + AF = 44$
5. $BC = 2MN = 16$ $AC = 2ML = 10$, so $NC = \frac{1}{2}AC = 5$ $AB = 2NL = 12$, so $MB = \frac{1}{2}AB = 6$ Perimeter = $BC + NC + MN + MB = 35$	6. $ST = 2(3.5) = 7$ (midsegment theorem) Let $x = RS = RT$ Perimeter $2x + 7 = 25$, so $x = 9$. $NT = \frac{1}{2}x = 4.5$ (def. of midpoint)

Regents Questions

1) JAN '17 [4] Ans: 4
2) AUG '17 [16] Ans: 4

4.9 Side Splitter Theorem

Practice Problems

1. $\frac{CB}{BA}=\frac{CE}{ET}$ $\frac{3}{10-3}=\frac{6}{x}$ $3x=42$ $x=14$	2. $\frac{12}{x}=\frac{16}{4}$ $16x=48$ $x=3$
3. $\frac{3}{5}=\frac{6}{BC}$ $3(BC)=30$ $BC=10$	4. $\frac{AD}{DB}=\frac{AE}{EC}$ $\frac{8}{10-8}=\frac{12}{x}$ $8x=24$ $x=3$
5. $\frac{3}{9}=\frac{x}{15}$ $9x=45$ $x=5$	6. $\frac{EB}{25}=\frac{12}{18}$ $18(EB)=300$ $EB\approx 16.7$
7. $\frac{MN}{NP}=\frac{MR}{RQ}$ $\frac{13}{8}=\frac{x}{42-x}$ $13(42-x)=8x$ $546-13x=8x$ $546=21x$ $x=26$ $MR = 26;\ RQ = 42 - 26 = 16$	8. $\frac{CD}{DA}=\frac{CE}{EB}$ $\frac{4}{10-4}=\frac{x+2}{4x-7}$ $4(4x-7)=6(x+2)$ $16x-28=6x+12$ $10x=40$ $x=4$ $CE = (4) + 2 = 6$

9. $CD = AD - AC = (2x+2) - (x-3) = x+5$

$$\frac{AB}{BE} = \frac{AC}{CD}$$
$$\frac{16}{20} = \frac{x-3}{x+5}$$
$$16x + 80 = 20x - 60$$
$$140 = 4x$$
$$35 = x$$
$$AC = x - 3 = 32$$

10. By the side splitter theorem,

$\frac{9}{6} = \frac{EL}{BE}$, or $\frac{6}{9} = \frac{BE}{EL}$.

By the angle bisector theorem,

$$\frac{BE}{x} = \frac{EL}{15}$$
$$x \cdot EL = 15 \cdot BE$$
$$x = 15 \cdot \frac{BE}{EL}$$

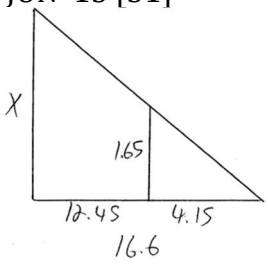
By substitution, $x = 15 \cdot \frac{6}{9} = 10$,
so $BW = 10$

11.

$\frac{5}{10} = \frac{12}{x}$ 24 miles

Regents Questions

1) JUN '15 [11] Ans: 3
2) AUG '15 [17] Ans: 4
3) JUN '16 [21] Ans: 2
4) JUN '17 [5] Ans: 4
5) JUN '17 [10] Ans: 2
6) AUG '17 [7] Ans: 4
7) JUN '18 [11] Ans: 2
8) JUN '18 [21] Ans: 4
9) AUG '18 [12] Ans: 2
10) AUG '18 [16] Ans: 3
11) JAN '19 [6] Ans: 2
12) JUN '15 [31]



$$\frac{1.65}{4.15} = \frac{x}{16.6}$$
$$x \approx 6.6$$

13) AUG '15 [27]

$$\frac{120}{230} = \frac{BC}{315}$$
$$BC \approx 164$$

14) JUN '16 [27]

$$\frac{3.75}{5} = \frac{4.5}{6}$$
$$22.5 = 22.5 \checkmark$$

$\overline{AB} \parallel \overline{CD}$ because $\overline{AB}$ divides the sides proportionately.

4.10 Centroid Median Theorem

Practice Problems

<table>
<tr><td colspan="2">1.
$SP = 2 \cdot PR = 24$ $\quad AT = 2 \cdot AR = 40$
$TM = PT + \frac{1}{2}PT = 42$ $\quad PY = \frac{1}{3}AY = 9$</td></tr>
<tr><td>2. $GC = 2(FG) = 24$ cm</td><td>3. $FD = \frac{1}{2}(AF) = 3$</td></tr>
<tr><td>4. $TA = TO + OA = 10 + 2(10) = 30$</td><td>5. $BP = \frac{2}{3}(BF) = \frac{2}{3}(18) = 12$</td></tr>
<tr><td>6. $FG = \frac{1}{3}(CF) = \frac{1}{3}(24) = 8$</td><td>7. $CR = 2(RF)$
$24 = 2(2x - 6)$
$24 = 4x - 12$
$36 = 4x$
$x = 9$</td></tr>
<tr><td>8. $BP = 2(PM)$
$7x + 4 = 2(2x + 5)$
$7x + 4 = 4x + 10$
$3x = 6$
$x = 2$
$PM = 2(2) + 5 = 9$</td><td>9. $QC = 2(CM)$
$5x = 2(x + 12)$
$5x = 2x + 24$
$3x = 24$
$x = 8$
$QM = QC + CM$
$= 5(8) + (8 + 12) = 60$</td></tr>
</table>

Regents Questions

1) JUN '18 [18] Ans: 1

5. RIGHT TRIANGLES AND TRIGONOMETRY

5.1 Pythagorean Theorem

PRACTICE PROBLEMS

1.

$$5^2+7^2=c^2$$
$$74=c^2$$
$$c=\sqrt{74}\approx 8.6$$

2.

$$24^2+b^2=26^2$$
$$576+b^2=676$$
$$b^2=100$$
$$b=10cm$$

3.

$x^2 + 19.5^2 = 20^2$
$x^2 + 380.25 = 400$
$x^2 = 19.75$
$x \approx 4.4$ ft.

4.

$$9^2+b^2=18^2$$
$$b^2=243$$
$$b\approx 15.6$$

5.

$$8^2+x^2=10^2 \qquad 4^2+y^2=10^2$$
$$64+x^2=100 \qquad 16+y^2=100$$
$$x^2=36 \qquad y^2=84$$
$$x=6 \qquad y=\sqrt{84}\approx 9.2$$

$y - x \approx 9.2 - 6 \approx 3.2\ ft.$

6.

$x^2 + (x+7)^2 = 13^2$
$x^2 + x^2 + 14x + 49 = 169$
$2x^2 + 14x - 120 = 0$
$2(x+12)(x-5) = 0$
$x = 5$ (reject $x = -12$)
Base $= 2x = 10$

7.

$x^2 + 7^2 = (x+1)^2$
$x^2 + 49 = x^2 + 2x + 1$
$2x = 48$
$x = 24$
Vertical bar is 25 inches

8.

$$(80-70)^2+(40-15)^2=c^2$$
$$10^2+25^2=c^2$$
$$725=c^2$$
$$c=\sqrt{725}\approx 26.93$$

$Area \approx 26.93 \times 6 \approx 162\ sq.\ ft.$

9.

$$6^2 + 8^2 = (DF)^2$$
$$100 = (DF)^2$$
$$DF = 10$$

Let $x = EG$. By angle bisector theorem,

$$\frac{x}{8-x} = \frac{6}{10}$$
$$6(8-x) = 10x$$
$$48 - 6x = 10x$$
$$48 = 16x$$
$$x = EG = 3$$

$$3^2 + 6^2 = (DG)^2$$
$$45 = (DG)^2$$
$$DG = \sqrt{45} \approx 6.7$$

REGENTS QUESTIONS

1) AUG '15 [11] Ans: 2

2) AUG '16 [8] Ans: 3

3) JAN '16 [32]

$$\frac{16}{9} = \frac{x}{20.6}$$
$$x \approx 36.6$$
$$36.6^2 + 20.6^2 = c^2$$
$$1763.92 = c^2$$
$$42 \approx c$$

4) AUG '17 [34]

$$x^2 + x^2 = 58^2$$
$$2x^2 = 3364$$
$$x = \sqrt{1682}$$
$$A = (\sqrt{1682} + 8)^2 \approx 2402.2$$

5.2 Congruent Right Triangles

Practice Problems

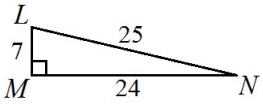
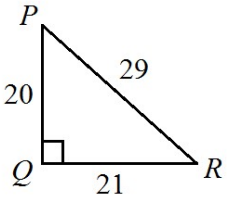
1. A	
2. (3)	3. (1)
4. Right triangles MAT and HTA (Given) $\overline{MT} \cong \overline{AH}$ (Given) $\overline{AT} \cong \overline{AT}$ (Reflexive Property) $\triangle MAT \cong \triangle HTA$ (HL) $\angle M \cong \angle H$ (CPCTC)	

Regents Questions

1) JUN '16 [7] Ans: 3

5.3 Trigonometric Ratios

Practice Problems

1. $\sin A = \frac{3}{5}, \cos A = \frac{4}{5}, \tan A = \frac{3}{4}$	2. $\sin B = \frac{15}{17}, \cos B = \frac{8}{17}, \tan B = \frac{15}{8}$
3. $\sin S$ and $\cos R$	4. $\tan B = \frac{8}{15} \approx 0.533$
5. $\sin x = \frac{28}{53} \approx 0.528$	6. $\cos A = \frac{16}{20} = 0.8$
7. $\sin N = \frac{7}{25}$ 	8. $\tan P = \frac{21}{20}$ 
9. $30^2 + 40^2 = c^2$ $2500 = c^2$ $c = 50$ $\sin B = \frac{30}{50} = \frac{3}{5}$	

Regents Questions

1) JUN '16 [15] Ans: 4
2) JAN '17 [14] Ans: 3
3) JAN '19 [17] Ans: 4

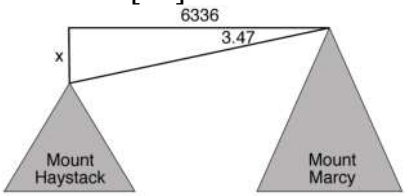
5.4 Use Trigonometry to Find a Side

Practice Problems

1. (3)	2. $\sin 15° = \frac{w}{10}$ $w = 10 \sin 15° \approx 2.6$
3. $\sin 32° = \frac{x}{30}$ $x = 30 \sin 32° \approx 15.9$ ft.	4. $\tan 62° = \frac{x}{15}$ $x = 15 \tan 62° \approx 28.2$ ft.
5. $\sin 57° = \frac{x}{8}$ $x = 8 \sin 57° \approx 6.7$ ft.	6. $\tan 32° = \frac{x}{25}$ $x = 25 \tan 32° \approx 15.6$ ft.
7. $\cos 65° = \frac{x}{5}$ $x = 5 \cos 65° \approx 2.1$ ft.	8. $\sin 48° = \frac{9}{x}$ $x = \frac{9}{\sin 48°} \approx 12$ ft.
9. $\tan 11° = \frac{400}{x}$ $x = \frac{400}{\tan 11°} \approx 2{,}058$ ft.	10. $\tan 58° = \frac{x}{6}$ $x = 6 \tan 58° \approx 9.60$ ft. $A = \frac{1}{2}bh \approx \frac{1}{2}(6)(9.60) \approx 28.8$ ft^2
11. $\tan 52° = \frac{50}{x}$ $x = \frac{50}{\tan 52°} \approx 39$ ft. from base to stake $\sin 52° = \frac{50}{x}$ $x = \frac{50}{\sin 52°} \approx 63$ ft. wire	12. $\sin 50° = \frac{x}{110}$ $x = 110 \sin 50° \approx 84$ ft. high $\cos 50° = \frac{x}{110}$ $x = 110 \cos 50° \approx 71$ ft. between the ropes
13. $\cos 72° = \frac{x}{10}$ $x = 10 \cos 72° \approx 3.09$ ft. ≈ 37 in. from base $\sin 72° = \frac{y}{10}$ $x = 10 \sin 72° \approx 9.51$ ft. ≈ 114 in. up wall	14. $\tan 28° = \frac{h}{200}$ (h = *cliff + lighthouse*) $h = 200 \tan 28° \approx 106.34$ ft. $\tan 18° = \frac{c}{200}$ (c = *cliff height alone*) $c = 200 \tan 18° \approx 64.98$ ft. $x = h - c \approx 106.34 - 64.98 \approx 41.4$ ft.

Regents Questions

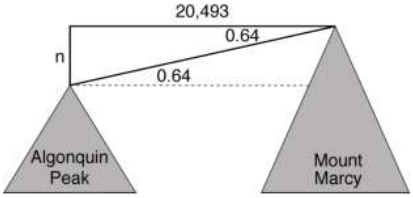
1) JUN '15 [5] Ans: 3
2) JUN '16 [11] Ans: 4
3) JAN '17 [7] Ans: 2
4) JAN '17 [12] Ans: 3
5) JUN '17 [21] Ans: 4
6) AUG '17 [19] Ans: 1
7) JAN '18 [4] Ans: 1
8) AUG '18 [6] Ans: 4
9) JAN '19 [13] Ans: 2
10) FALL '14 [13]



$\tan 3.47 = \frac{M}{6336}$

$M \approx 384$

4960 + 384 = 5344



$\tan 0.64 = \frac{A}{20493}$

$A \approx 229$

5344 – 229 = 5115

11) FALL '14 [22]

x represents the distance between the lighthouse and the canoe at 5:00; *y* represents the distance between the lighthouse and the canoe at 5:05.

$\tan 6 = \frac{112 - 1.5}{x}$

$x \approx 1051.3$

$\tan 55 = \frac{112 - 1.5}{y}$

$y \approx 77.4$

$\frac{1051.3 - 77.4}{5} \approx 195$

12) AUG '15 [32]

$\tan 7 = \frac{125}{AC}$

$AC = \frac{125}{\tan 7} \approx 1018.0$

$\tan 16 = \frac{125}{DC}$

$DC = \frac{125}{\tan 16} \approx 435.9$

$AD = AC - DC$

$\approx 1018.0 - 435.9 \approx 582$

13) JAN '16 [29]

$\sin 70 = \frac{30}{x}$

$x \approx 32$

14) JAN '16 [36]

$\tan 52.8 = \frac{h}{x}$

$h = x \tan 52.8 \approx 1.32x$

$\tan 34.9 = \frac{h}{x + 8}$

$h = (x + 8) \tan 34.9 \approx 0.70(x + 8)$

$1.32x = 0.7(x + 8)$

$1.32x = 0.7x + 5.6$

$0.62x = 5.6$

$x \approx 9.0$

$\tan 52.8 \approx \frac{h}{9}$

$h \approx 9 \tan 52.8 \approx 11.86$

11.86 + 1.7 ≈ 13.6

15) AUG '16 [31]

$\sin 75 = \frac{15}{x}$

$x = \frac{15}{\sin 75}$

$x \approx 15.5$

16) JUN '17 [36]

$\tan 15 = \frac{6250}{x}$

$x \approx 23{,}325.3$

$\tan 52 = \frac{6250}{y}$

$y \approx 4{,}883.0$

23,325.3 – 4,883.0 = 18,442.3

Plane traveled 18,442 ft. in 1 min.

$\frac{18442\,\text{ft}}{1\,\text{min}} \cdot \frac{1\,\text{mile}}{5280\,\text{ft}} \cdot \frac{60\,\text{min}}{1\,\text{hr}} \approx 210\,\text{mph}$

17) JAN '18 [34]

$$\cos 54 = \frac{4.5}{HI}$$

$$HI \approx 7.7 \text{ mi}$$

$$\tan 54 = \frac{IM}{4.5}$$

$$IM \approx 6.2 \text{ mi}$$

18) JUN '18 [33]

$$\tan 72° = \frac{ST}{400}$$

$$ST = 400 \tan 72° \approx 1231.07$$

$$\sin 55° = \frac{ST}{CT} \approx \frac{1231.07}{CT}$$

$$CT \approx \frac{1231.07}{\sin 55°} \approx 1503$$

19) AUG '18 [33]

$\text{m}\angle AGH = 36$

$\tan 36 = \frac{HA}{10}$

$HA = 10 \tan 36 \approx 7.2654$

$HA = FG = DE \approx 7.3$

$AG = \sqrt{(HA)^2 + (HG)^2} =$

$\sqrt{7.2654^2 + 10^2} \approx 12.3607$

$AC = 3 \times 12.3607 \approx 37$

20) JAN '19 [34]

$\sin 4.76° = \frac{6 \cdot 3}{x}$

$x = \frac{18}{\sin 4.76°} \approx 216.914$ in. ≈ 18.1 ft.

$\tan 4.76° = \frac{18}{y}$

$y = \frac{18}{\tan 4.76°} \approx 216.166$ in.

$d = 216.166 - 16 =$

200.166 in. ≈ 16.7 ft.

5.5 Use Trigonometry to Find an Angle

Practice Problems

1. $\sin x = \frac{3}{7}$ $x = \sin^{-1}\left(\frac{3}{7}\right) \approx 25.4°$	2. $\tan A = \frac{11.2}{18.3}$ $A = \tan^{-1}\left(\frac{11.2}{18.3}\right) \approx 31.5°$
3. $\sin x = \frac{30}{50}$ $x \approx 37°$	4. $\sin A = \frac{8}{12}$ $A \approx 42°$
5. $\cos x = \frac{6}{28}$ $x \approx 78°$	6. $\sin A = \frac{10}{16}$ $m\angle A \approx 38.7°$ $m\angle B = 90 - m\angle A \approx 51.3°$
7. $\tan x = \frac{420}{2000}$ $x \approx 12°$	8. $\tan x = \frac{350}{1000}$ $x \approx 19°$
9. $12^2 + 16^2 = r^2$ $r = 20$ $s = 50 - 20 = 30$ $\sin x = \frac{16}{30}$ $x \approx 32°$	
10. 36 – 28 = 8 ft $\sin x = \frac{8}{12}$ $x \approx 41.8°$	
11. $\tan x = \frac{6}{4}$ $x \approx 56°$ $\sin 56° = \frac{b}{15}$ $b = 15 \sin 56° \approx 12$ ft.	

Regents Questions

1) FALL '14 [1] Ans: 1
2) JAN '16 [16] Ans: 3
3) JUN '17 [13] Ans: 1
4) AUG '17 [15] Ans: 1
5) JUN '18 [6] Ans: 2
6) AUG '18 [9] Ans: 1
7) JUN '15 [28]
$$\sin x = \frac{4.5}{11.75}$$
$$x \approx 23$$
8) JUN '16 [30]
$$\tan x = \frac{10}{4}$$
$$x \approx 68$$
9) AUG '16 [34]
$$\tan x = \frac{12}{75}$$
$$x \approx 9.09$$
$$\tan y = \frac{72}{75}$$
$$y \approx 43.83$$
$$y - x \approx 34.7°$$
10) JAN '18 [31]
$$\cos x = \frac{6}{18}$$
$$x \approx 71$$

5.6 Special Triangles

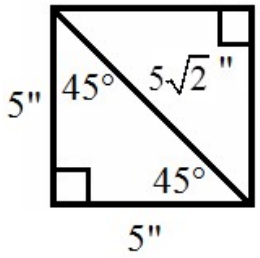
Practice Problems

1. 30-60-90 triangle with a factor of 12. $x = 12\sqrt{3} \approx 21$

2. $5\sqrt{2}$ inches

   Each side is 5 inches.

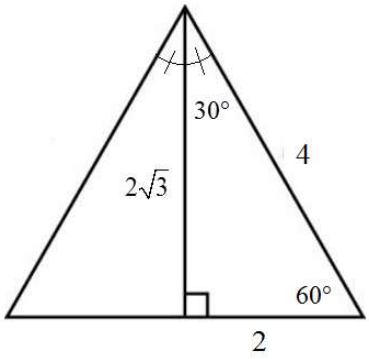
   45-45-90 triangle with factor of 5.



3. 4 units

   The altitude of an equilateral triangle is also the angle bisector.

   30-60-90 triangle with factor of 2.



Regents Questions

1) JAN '17 [9] Ans: 2

5.7 Cofunctions

Practice Problems

1. $\text{m}\angle B = 90 - 25 = 65°$	2. $72 + x = 90$ $x = 18°$
3. $x + 15 + x - 5 = 90$ $2x + 10 = 90$ $2x = 80$ $x = 40$	4. $2x - 1 + 3x + 6 = 90$ $5x + 5 = 90$ $5x = 85$ $x = 17$
5. $x + 20 + x = 90$ $2x + 20 = 90$ $2x = 70$ $x = 35$	6. $x - 3 + 2x + 6 = 90$ $3x + 3 = 90$ $3x = 87$ $x = 29$
7. $2x + 20 + 40 = 90$ $2x + 60 = 90$ $2x = 30$ $x = 15$	8. $2x - 25 + 55 = 90$ $2x + 30 = 90$ $2x = 60$ $x = 30$

Regents Questions

1) JUN '15 [12] Ans: 4
2) AUG '15 [4] Ans: 1
3) JAN '16 [9] Ans: 4
4) AUG '16 [6] Ans: 1
5) JUN '17 [3] Ans: 3
6) AUG '17 [21] Ans: 4
7) JUN '18 [8] Ans: 1
8) AUG '18 [24] Ans: 2
9) JAN '19 [22] Ans: 1
10) FALL '14 [7]

$$2x + 0.1 = 4x - 0.7$$
$$0.8 = 2x$$
$$0.4 = x$$

A and B are complementary angles, and cofunctions of complementary angles are equal.

11) FALL '14 [20]
The acute angles in a right triangle are always complementary. The sine of any acute angle is equal to the cosine of its complement.

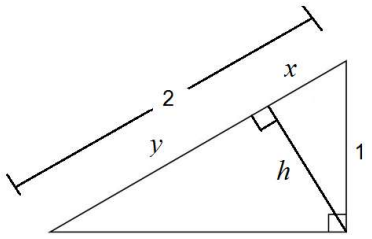
12) JUN '16 [28]
$R = 90 - 73 = 17°$
Cofunctions of complementary angles are equal.

13) JAN '17 [27]
Yes, because 28º and 62º angles are complementary. The sine of an angle equals the cosine of its complement.

14) JAN '18 [27]
Since angles A and B are complentary and sine and cosine are cofunctions, $\sin A = \cos B$. Therefore, when $\sin A$ increases, $\cos B$ increases.

5.8 Mean Proportional

Practice Problems

1. $x^2 = 3 \cdot 9 = 27$ $x = \sqrt{27} = 3\sqrt{3}$	2. $x^2 = 12 \cdot 3 = 36$ $x = \sqrt{36} = 6$
3. $AD = 16 - 7 = 9$ $x^2 = 7 \cdot 9 = 63$ $x = \sqrt{63} = 3\sqrt{7}$	4. $x^2 = 12 \cdot 2 = 24$ $x = \sqrt{24} = 2\sqrt{6}$
5. $x^2 = 4 \cdot 7 = 28$ $x = \sqrt{28} = 2\sqrt{7}$	6. $x^2 = 8 \cdot 18 = 144$ $x = 12$
7. $6^2 = x(x + 5)$ $36 = x^2 + 5x$ $x^2 + 5x - 36 = 0$ $(x + 9)(x - 4) = 0$ $x = 4$ (reject $x = -9$)	8. $10^2 = x(x + 21)$ $100 = x^2 + 21x$ $x^2 + 21x - 100 = 0$ $(x + 25)(x - 4) = 0$ $x = 4$ (reject $x = -25$)
9. Let $x = AD$ $4^2 = x(x + 6)$ $x^2 + 6x - 16 = 0$ $(x + 8)(x - 2) = 0$ $x = 2$ (reject $x = -8$) $AC = x + (x + 6) = 10$	10. $x^2 = 4 \cdot 5 = 20$ $x = \sqrt{20} = 2\sqrt{5}$ $y^2 = 9 \cdot 4 = 36$ $y = \sqrt{36} = 6$ $z^2 = 9 \cdot 5 = 45$ $z = \sqrt{45} = 3\sqrt{5}$
11. $x^2 = 24 \cdot 6 = 144$ $x = \sqrt{144} = 12$ $y^2 = 30 \cdot 6 = 180$ $y = \sqrt{180} = 6\sqrt{5}$ $z^2 = 24 \cdot 30 = 720$ $z = \sqrt{720} = 12\sqrt{5}$	12.  (a) $1^2 = 2x$ $1 = 2x$ $x = \frac{1}{2}$ (b) $y = 2 - \frac{1}{2} = \frac{3}{2}$ $h^2 = xy = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$ $h = \sqrt{\frac{3}{4}}$

Regents Questions

1) JAN '16 [22] Ans: 2
2) JUN '16 [13] Ans: 2
3) AUG '16 [10] Ans: 2
4) AUG '17 [18] Ans: 2
5) JAN '18 [23] Ans: 2
6) JUN '18 [23] Ans: 1
7) AUG '18 [7] Ans: 3
8) AUG '18 [20] Ans: 2
9) JAN '19 [10] Ans: 3
10) JUN '15 [34]

$\sqrt{0.55^2 - 0.25^2} \approx 0.49$

No, $0.49^2 = 0.25y$

$0.9604 = y$

$0.9604 + 0.25 < 1.5$

11) JUN '17 [29]
If an altitude is drawn to the hypotenuse of a triangle, it divides the triangle into two right triangles that are similar to each other and to the original triangle.

5.9 SAS Sine Formula for Area of a Triangle

Practice Problems

1. $A = \frac{1}{2} \cdot 6 \cdot 8 \cdot \frac{1}{4} = 6$	2. $A = \frac{1}{2} \cdot 12 \cdot 15 \cdot \sin 150 = 45$
3. $A = \frac{1}{2} \cdot 7 \cdot 10 \cdot \sin 25 \approx 14.8$	4. $\text{m}\angle C = 180 - 2 \cdot 75 = 30$ $A = \frac{1}{2} \cdot 10 \cdot 10 \cdot \sin 30 = 25$
5. $A = \frac{1}{2} \cdot 14 \cdot 16 \cdot \sin 30 = 56$	6. $A = \frac{1}{2} \cdot 11 \cdot 13 \cdot \sin 70 \approx 67$
7. $A = \frac{1}{2} \cdot 4 \cdot 4 \cdot \sin 30° = 4$	8. $A = \frac{1}{2} \cdot 4 \cdot 5 \cdot \sin 59° \approx 8.6$
9. $A = \frac{1}{2} \cdot 16 \cdot 21 \cdot \sin 58 \approx 142.5$	10. $A = \frac{1}{2} \cdot 12 \cdot 8 \cdot \sin 40° \approx 30.9$
11. $A = \frac{1}{2} \cdot 12 \cdot 31 \cdot \sin 62 \approx 164.2$	12. Vertex angle = $180 - 2(50) = 80°$ $A = \frac{1}{2}(20.4)(20.4) \sin 80 \approx 204.9$
13. $\text{m}\angle R = 180 - (38 + 17) = 125°$ $A = \frac{1}{2}(15)(31.6) \sin 125 \approx 194$	14. $12 = \frac{1}{2} \cdot 8 \cdot b \cdot \sin 30°$ $12 = 4b \cdot \sin 30°$ $3 = b \cdot \sin 30°$ $b = \frac{3}{\sin 30°} = 6$
15. $42 = \frac{1}{2} \cdot 24 \cdot b \cdot \sin 30°$ $42 = 12b \cdot \sin 30°$ $3.5 = b \cdot \sin 30°$ $b = \frac{3.5}{\sin 30°} = 7$	16. $12 = \frac{1}{2} \cdot 6 \cdot c \cdot \sin 30°$ $12 = 3c \cdot \sin 30°$ $4 = c \cdot \sin 30°$ $c = \frac{4}{\sin 30°} = 8$

6. QUADRILATERALS

6.1 Properties of Quadrilaterals

Practice Problems

1. (4) trapezoid

2. (1) rhombus

3. (3) rhombus

4. (3) the rhombus and square

5. (2) An isosceles trapezoid has a pair of parallel bases and a pair of congruent legs, but may not be a parallelogram.

6. In a parallelogram, diagonals bisect each other, so $AM = 10$.

7. In a parallelogram, opposite angles are congruent, so
$6x - 30 = 4x + 10$
$2x = 40$
$x = 20$
$m\angle A = 6(20) - 30 = 90$
$ABCD$ is a rectangle because it is a parallelogram with a right angle.
Note: It is sufficient to show that one angle of a parallelogram is a right angle. If one angle in a parallelogram is a right angle, then all four angles must be right angles.

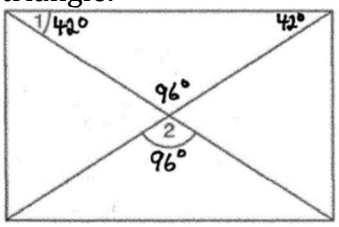
8. In a parallelogram, diagonals bisect each other, so $AE = 5$.
In a rhombus, diagonals are perpendicular, so $\triangle AEB$ is a right triangle with a right angle at E.
$(BE)^2 = (AB)^2 - (AE)^2$
$(BE)^2 = 8^2 - 5^2 = 39$
$BE = \sqrt{39} \approx 6.24$
$BD = 2 \cdot BE \approx 12.5$

9. Since $ABCD$ is a rhombus, all sides are congruent, so $AB = AD = 24$.
Diagonals of a rhombus are perpendicular, so $\triangle AEB$ is a right triangle with a right angle at E.
$\sin 30 = \frac{BE}{24}$
$BE = 24 \sin 30 = 12$
Diagonals bisect each other, so
$DE = BE = 12$

10. Since $ABCD$ is a rhombus, the diagonals are angle bisectors, so
$m\angle ABD = \frac{1}{2} m\angle ABC = 30$.
Diagonals of a rhombus are perpendicular, so $\triangle AEB$ is a right triangle with a right angle at E.
$\sin 30 = \frac{18}{AB}$
$AB = \frac{18}{\sin 30} = 36$
All sides of a rhombus are congruent, so
$DC = AB = 36$.

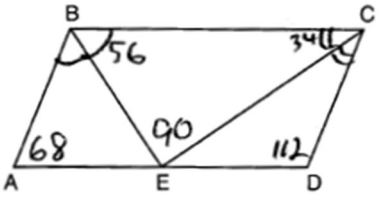
11. $AE = \frac{1}{2}AC = 9 \quad BE = \frac{1}{2}BD = 12$
(diagonals bisect each other)
$m\angle AEB = 90°$ ($\perp$ diagonals)
$(AB)^2 = 9^2 + 12^2 = 225$
$AB = \sqrt{225} = 15$
(Pythagorean Thm.)

12. Label E at the intersection of diagonals.
$AE = \frac{1}{2}AC = 6$
$(DE)^2 + 6^2 = 10^2$
$(DE)^2 = 64$
$DE = 8$
$DB = 2 \cdot DE = 16$

13. $m\angle ABC = 180 - 100 = 80°$ (consecutive angles supplementary) $m\angle DBC = \frac{1}{2}80 = 40°$ (diagonals are angle bisectors)	14. $m\angle 1 + m\angle 2 = 180 - 120 = 60°$ (consecutive angles supplementary) $m\angle 2 = 60 - 45 = 15°$
15. Label E at the intersection of diagonals. $m\angle AEM = 90°$ ($\perp$ diagonals) $m\angle AMT = 90 - 12 = 78°$	16. $\angle 1$ is a base angle of an isosceles triangle.  $m\angle 2 = 96$

Regents Questions

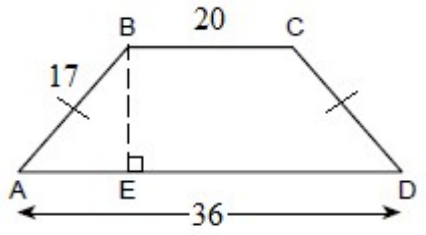
1) AUG '15 [1] Ans: 2
2) AUG '15 [8] Ans: 3
3) JAN '16 [3] Ans: 3
4) AUG '16 [24] Ans: 1
5) JAN '17 [16] Ans: 1
6) JUN '17 [11] Ans: 4
7) AUG '17 [8] Ans: 4
8) JAN '18 [2] Ans: 2
9) JAN '18 [19] Ans: 4
10) AUG '18 [13] Ans: 4
11) JAN '19 [7] Ans: 2
12) JAN '19 [12] Ans: 2
13) JUN '15 [26]
Opposite angles in a parallelogram are congruent, so $m\angle O = 118°$.
The interior angles of a triangle equal 180°. 180 – (118 + 22) = 40
14) AUG '17 [26]
The four small triangles are 8-15-17 triangles. $4 \times 17 = 68$
15) AUG '18 [26]
Ans: 90°



16) JAN '19 [26]
$m\angle ADC = m\angle B = 118°$ (opp. $\angle$'s of $\square$ are $\cong$)
$m\angle DGF = m\angle ADC = 118°$ (alt. int. $\angle$'s of $\parallel$ lines $\overline{FG}$ and $\overline{EDC}$)
$m\angle GFH = m\angle AHC - m\angle DGF$ ($\angle AHC$ is an ext. angle of $\triangle FGH$)
$= 138° - 118° = 20°$

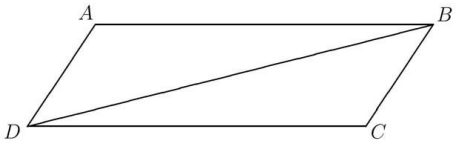
6.2 Trapezoids

Practice Problems

1. (a) Diagonals are congruent, so $BD = 25$. (b) $m\angle D = m\angle C = 180 - 75 = 105$.	2. $GT = \frac{40-24}{2} = 8$ $(AG)^2 = 10^2 - 8^2 = 36$ $AG = \sqrt{36} = 6$ $LF = AG = 6$ (altitudes are congruent)
3. Draw altitude $\overline{CE}$. $ED = \frac{26-1}{2} = 7$ $(CE)^2 = 25^2 - 7^2$ $CE = \sqrt{576} = 24$	4.  $AE = \frac{36-20}{2} = 8$ $(BE)^2 = 17^2 - 8^2 = 225$ $BE = \sqrt{225} = 15$
5. Draw altitude $\overline{AT}$. $\triangle RAT$ is an isosceles right triangle with legs of 6. $(RA)^2 = 6^2 + 6^2$ $RA = \sqrt{72} = 6\sqrt{2}$	6. $\triangle RST$ They share the same base, $\overline{RS}$, and congruent altitudes (since $\overline{VT} \parallel \overline{RS}$).

6.3 Parallelogram Proofs

Practice Problems

<table>
<tr><td>1. Yes, one pair of opposite sides are both parallel and congruent.</td><td>2. Yes, both pairs of opposite sides are congruent.</td></tr>
<tr><td colspan="2">3. (3) Diagonals are perpendicular.</td></tr>
<tr><td>4.
 

$ABCD$ is a parallelogram (Given)
$\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{CB}$ (opposite sides of parallelograms are congruent)
$\overline{BD} \cong \overline{BD}$ (Reflexive Property)
$\triangle ABD \cong \triangle CDB$ (SSS)</td><td>5. $\triangle AOB \cong \triangle COD$ (Given)
$\overline{AB} \cong \overline{CD}$ (CPCTC)
$\angle OAB \cong \angle OCD$ (CPCTC)
$\overline{AB} \parallel \overline{CD}$ (if two lines cut by a transversal form congruent alternate interior angles, then they are parallel)
$ABCD$ is a parallelogram (if a quadrilateral has a pair of opposite sides that are both parallel and congruent, then it is a parallelogram)</td></tr>
<tr><td>6. $ABCD$ is a parallelogram, $DF = EB$ (Given)
$\overline{AEB} \parallel \overline{DFC}$ (opposite sides of a parallelogram are parallel)
$\overline{AB} \cong \overline{DC}$ (opposite sides of a parallelogram are congruent)
$AB = DC$ (definition of congruence)
$AB - EB = DC - DF$
(Subtraction Postulate)
$AE = AB - EB, FC = DC - DF$
(Partition Postulate)
$AE = FC$ (substitution)
$\overline{AE} \cong \overline{FC}$ (definition of congruence)
$AECF$ is a parallelogram (if a quadrilateral has a pair of opposite sides that are both parallel and congruent, then it is a parallelogram)</td><td>7. $ABCD$ is a parallelogram, $CEBF$ is a rhombus (Given)
$\overline{BE} \cong \overline{CE}$ (all sides of a rhombus are congruent)
$BE = CE$ (definition of congruence)
$2 \cdot BE = 2 \cdot CE$ (Multiplication Property)
$BD = 2 \cdot BE, AC = 2 \cdot CE$ (diagonals of a parallelogram bisect each other)
BD = AC (Substitution)
$\overline{BD} \cong \overline{AC}$ (definition of congruence)
$ABCD$ is a rectangle (if the diagonals of a parallelogram are congruent, then it is a rectangle)</td></tr>
</table>

Regents Questions

1) JUN '15 [13] Ans: 4
2) JUN '16 [9] Ans: 1
3) AUG '16 [7] Ans: 3
4) JAN '17 [5] Ans: 4
5) JUN '17 [20] Ans: 2
6) AUG '17 [14] Ans: 3
7) JUN '18 [13] Ans: 4
8) AUG '15 [28]
Parallelogram $ABCD$, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E (Given).
$\overline{DC} \parallel \overline{AB}$; $\overline{DA} \parallel \overline{CB}$ (opposite sides of a parallelogram are parallel).
$\angle ACD \cong \angle CAB$ (alternate interior angles formed by parallel lines and a transversal are congruent).
9) AUG '15 [35]
Parallelogram $ABCD$, $\overline{BE} \perp \overline{CED}$, $\overline{DF} \perp \overline{BFC}$, $\overline{CE} \cong \overline{CF}$ (Given).
$\angle BEC \cong \angle DFC$ (perpendicular lines form right angles, which are congruent). $\angle FCD \cong \angle BCE$ (reflexive property).
$\triangle BEC \cong \triangle DFC$ (ASA).
$\overline{BC} \cong \overline{CD}$ (CPCTC).
$ABCD$ is a rhombus (a parallelogram with consecutive congruent sides is a rhombus).
10) JAN '16 [35]
Parallelogram $ANDR$ with $\overline{AW}$ and $\overline{DE}$ bisecting $\overline{NWD}$ and $\overline{REA}$ at points W and E (Given).
$\overline{AN} \cong \overline{RD}$, $\overline{AR} \cong \overline{DN}$ (Opposite sides of a parallelogram are congruent).
(Opposite angles of a parallelogram are congruent).
$RE = \frac{1}{2}AR$, $NW = \frac{1}{2}DN$, so $\overline{RE} \cong \overline{NW}$ (Definition of bisect and division property).
$\triangle ANW \cong \triangle DRE$ (SAS).
$AE = \frac{1}{2}AR$, $WD = \frac{1}{2}DN$, so $\overline{AE} \cong \overline{WD}$ (Definition of bisect and division property).
$\overline{AER} \parallel \overline{NWD}$ (Opposite sides of a parallelogram are parallel).
$AWDE$ is a parallelogram (A quadrilateral with a pair of opposite sides that are congruent and parallel is a parallelogram).
11) JUN '16 [33]
Parallelogram $ABCD$, $\overline{EFG}$, and diagonal $\overline{DFB}$ (Given);
$\angle DFE \cong \angle BFG$ (vertical angles);
$\overline{AD} \parallel \overline{CB}$ (opposite sides of a parallelogram are parallel);
$\angle EDF \cong \angle GBF$ (alternate interior angles are congruent);
$\triangle DEF \sim \triangle BGF$ (AA)

12) JUN '16 [35]
Quadrilateral *ABCD* with diagonals $\overline{AC}$ and $\overline{BD}$ that bisect each other, and $\angle 1 \cong \angle 2$ (Given); quadrilateral *ABCD* is a parallelogram (if the diagonals of a quadrilateral bisect each other, it is a parallelogram); quadrilateral *ABCD* is a rhombus (if a diagonal of a parallelogram bisects its angle, it is a rhombus); $\overline{AD} \cong \overline{DC}$ (the sides of a rhombus are congruent); $\triangle ACD$ is an isosceles triangle (definition of isosceles triangle); $\overline{AE} \perp \overline{BE}$ (the diagonals of a rhombus are perpendicular); $\angle BEA$ is a right angle (perpendicular lines form a right angle); $\triangle AEB$ is a right triangle (definition of right triangle).

13) JAN '17 [35]
Quadrilateral *ABCD*, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \parallel \overline{CD}$, and $\overline{BF}$ and $\overline{DE}$ are perpendicular to diagonal $\overline{AC}$ at points *F* and *E* (given). $\angle AED$ and $\angle CFB$ are right angles (perpendicular lines form right angles). $\angle AED \cong \angle CFB$ (All right angles are congruent). *ABCD* is a parallelogram (A quadrilateral with one pair of sides congruent and parallel is a parallelogram). $\overline{AD} \parallel \overline{BC}$ (Opposite sides of a parallelogram are parallel). $\angle DAE \cong \angle BCF$ (Parallel lines cut by a transversal form congruent alternate interior angles). $\overline{DA} \cong \overline{BC}$ (Opposite sides of a parallelogram are congruent). $\triangle ADE \cong \triangle CBF$ (AAS). $\overline{AE} \cong \overline{CF}$ (CPCTC).

14) JAN '18 [25]
Parallelogram *ABCD,* diagonal $\overline{AC}$ (Given); $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{BC}$ (opposite sides of a parallelogram are congruent); $\overline{AC} \cong \overline{AC}$ (Reflexive property); $\triangle ABC \cong \triangle CDA$ (SSS)

15) JUN '18 [35]
Parallelogram *ABCD*, $\overline{BF} \perp \overline{AFD}$, and $\overline{DE} \perp \overline{BEC}$ (Given); $\overline{BC} \parallel \overline{AD}$ (opp sides of a parallelogram are parallel); $\overline{BE} \parallel \overline{FD}$ (parts of || lines are ||); $\overline{BF} \parallel \overline{DE}$ (two lines ⊥ to same line are ||); *BEDF* is a parallelogram (quadrilateral with both pairs of opposite sides || is a parallelogram); $\angle DEB$ is a right angle (⊥ lines form right angles); *BEDF* is a rectangle (parallelogram with a right angle is a rectangle)

16) JAN '19 [35]
(Givens omitted); $\overline{HF} \cong \overline{HF}$ (Reflexive property); $\overline{HF} + \overline{CF} \cong \overline{HF} + \overline{AH}$, so $\overline{AF} \cong \overline{CH}$ (Segment addition); *ABCD* is a $\square$ (Both pairs of opp. sides are ≅); $\overline{AB} \parallel \overline{CD}$ (Opp. sides of a $\square$ are parallel); $\angle BAC \cong \angle DCA$ (Alternate interior angles); $\triangle EAF \cong \triangle GCH$ (SAS); $\overline{EF} \cong \overline{GH}$ (CPCTC)

7. COORDINATE GEOMETRY

7.1 Forms of Linear Equations

PRACTICE PROBLEMS

1. No. $(3) = 3(-2) + 15?$ $3 \neq 9$	2. $y = mx + b$ $y = -4x + 5$
3. $y - y_1 = m(x - x_1)$ $y + 2 = -3(x - 1)$	4. $m = \frac{-5+3}{5+2} = -\frac{2}{7}$ $y - y_1 = m(x - x_1)$ $y + 3 = -\frac{2}{7}(x + 2)$
5. $m = \frac{5-3}{8-1} = \frac{2}{7}$ $y - 3 = \frac{2}{7}(x - 1)$	6. $m = \frac{4-0}{5-(-5)} = \frac{2}{5}$ $y - 4 = \frac{2}{5}(x - 5)$
7. $-2x + y = -5$ $2x - y = 5$	8. $4y = 4\left(\frac{3}{4}x + \frac{1}{2}\right)$ $4y = 3x + 2$ $-3x + 4y = 2$ $3x - 4y = -2$

7.2 Parallel and Perpendicular Lines

Practice Problems

1. equation (1)
$2y+2x=6$
$2y=-2x+6$
$y=-x+3$

2. equation (1)
$-3y=2x+5$
$-2x-3y=5$
$-2(-2x-3y=5)$
$4x+6y=-10$

3. equation (2)

4. equation (1)
$x-5y=25$
$-5y=-x+25$
$y=\frac{1}{5}x-5$

5. $y=-2x+2$

6. $y=-2x$

7. $2y-x=8$ $\quad (-1)=\frac{1}{2}(4)+b$
$2y=x+8$ $\quad b=-3$
$y=\frac{1}{2}x+4$
$m=\frac{1}{2}$ $\quad y=\frac{1}{2}x-3$

8. $m=-3$
$(12)=-3(-9)+b$
$b=-15$
$y=-3x-15$

9. $3y=6x+3$ $\quad (4)=-\frac{1}{2}(2)+b$
$y=2x+1$ $\quad b=5$
$m=-\frac{1}{2}$ $\quad y=-\frac{1}{2}x+5$

10. The slope of $y=2x+3$ is 2. The slope of $2y+x=6$ is $-\frac{1}{2}$. Since the slopes are opposite reciprocals, the lines are perpendicular.

11. The slope of $x+2y=4$ is $-\frac{1}{2}$. The slope of $4y-2x=12$ is $\frac{1}{2}$. Since the slopes are neither equal nor opposite reciprocals, the lines are neither parallel nor perpendicular.

12. $\frac{x-1}{4}=\frac{-3}{8}$
$2(x-1)=-3$
$2x-2=-3$
$x=-\frac{1}{2}$

Regents Questions

1) JUN '15 [9] Ans: 1
2) AUG '15 [10] Ans: 1
3) JAN '16 [2] Ans: 4
4) JUN '16 [14] Ans: 4
5) JAN '17 [1] Ans: 3
6) JUN '17 [19] Ans: 2
7) JAN '18 [20] Ans: 1
8) JUN '18 [12] Ans: 2
9) AUG '18 [11] Ans: 1
10) JAN '19 [25]
$y=\frac{(2x-7)}{3}, m=\frac{2}{3}$
$y-6=\frac{2}{3}(x-2)$

7.3 Distance Formula

Practice Problems

1. $d=\sqrt{(6+2)^2+(-3-3)^2}$
$=\sqrt{64+36}=\sqrt{100}=10$

2. $d=\sqrt{(8-3)^2+(10-5)^2}$
$=\sqrt{25+25}=\sqrt{50}=5\sqrt{2}$

3. $d=\sqrt{(7+1)^2+(4-9)^2}$
$=\sqrt{64+25}=\sqrt{89}$

4. $d=\sqrt{(1-5)^2+(6-3)^2}$
$=\sqrt{16+9}=\sqrt{25}=5$

5. $d=\sqrt{(7-3)^2+(2+4)^2}$
$=\sqrt{16+36}=\sqrt{52}=2\sqrt{13}$

6. $d=\sqrt{(6-3)^2+(-1-8)^2}$
$=\sqrt{9+81}=\sqrt{90}=3\sqrt{10}$

7. $d=\sqrt{(4+3)^2+(25-1)^2}$
$=\sqrt{49+576}=\sqrt{625}=25$

8. $d=\sqrt{(146+4)^2+(52-2)^2}$
$=\sqrt{22{,}500+2{,}500}=\sqrt{25{,}000}$
$=50\sqrt{10}$

9. $m_{\perp}=-5$

Equation of $\perp$ line: $y+2=-5(x-6)$, $y=-5x+28$

Solve system for x: $-5x+28=\frac{1}{5}x+2$, $x=5$

Solve for y: $y=\frac{1}{5}(5)+2$, $y=3$

Distance for (6,–2) to (5,3): $d=\sqrt{(5-6)^2+(3+2)^2}$, $=\sqrt{1+25}=\sqrt{26}$

Regents Questions

1) JUN '15 [3] Ans: 3
2) JAN '16 [15] Ans: 2

7.4 Midpoint Formula

Practice Problems

1. $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ $=\left(\frac{-2+6}{2},\frac{3-3}{2}\right)=(2,0)$	2. $\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ $=\left(\frac{3+8}{2},\frac{5+10}{2}\right)=\left(5\frac{1}{2},7\frac{1}{2}\right)$
3. The third terms of the arithmetic sequences x: 1, 3, … and y: –5, 5, … are 5 and 15, so $B(5,15)$.	4. M is the midpoint of BC. Sequences x: 4, 0, … and y: –3, –2, … give us third terms of –4 and –1, so $C(-4,-1)$.
5. $\left(\frac{-5+3}{2},\frac{1+5}{2}\right)=(-1,3)$	6. $\left(\frac{1+10}{2},\frac{3+7}{2}\right)=\left(\frac{11}{2},5\right)$
7. x: 1, 3.5, 6 and y: 8, 2, –4, so $R(6,-4)$	8. x: –1, 2, 5 and y: 5, 3, 1, so (5,1)

Regents Questions

1) JAN '19 [30]
No. (a) If $\overline{EG}$ is a median, then G is the midpoint of $\overline{DF}$. But midpoint of $\overline{DF}$ is $\left(\frac{1+4}{2},\frac{-1+2}{2}\right)=(2.5,0.5)$,
or (b) If $\overline{EG}$ is a median, then $DG = GF$. But, $DG=\sqrt{2^2+2^2}=\sqrt{8}$ and $GF=\sqrt{1^2+1^2}=\sqrt{2}$.

7.5 Perpendicular Bisectors

Practice Problems

1. Midpoint is $\left(\frac{3+9}{2},\frac{5+17}{2}\right)=(6,11)$

   $m=\frac{17-5}{9-3}=\frac{12}{6}=2 \qquad m_{\perp}=-\frac{1}{2}$

   Equation of perpendicular bisector is

   $y-11=-\frac{1}{2}(x-6)$

2. Midpoint is $\left(\frac{-2+6}{2},\frac{3-3}{2}\right)=(2,0)$

   $m=\frac{-3-3}{6+2}=\frac{-6}{8}=-\frac{3}{4} \qquad m_{\perp}=\frac{4}{3}$

   Equation of perpendicular bisector is

   $y=\frac{4}{3}(x-2)$

3. Midopoint is $\left(\frac{2+8}{2},\frac{6+12}{2}\right)=(5,9)$

   $m=\frac{12-6}{8-2}=\frac{6}{6}=1 \qquad m_{\perp}=-1$

   Equation of perpendicular bisector is

   $y-9=-(x-5)$

   $y-9=-x+5$

   $y=-x+14$

   y-intercept is 14

4. Midopoint is $\left(\frac{-4+2}{2},\frac{5+5}{2}\right)=(-1,5)$

   The segment is horizontal with a 0 slope.

   Its perpendicular bisector is vertical, with an equation of $x=-1$.

5. $M=\left(\frac{4+8}{2},\frac{2+6}{2}\right)=(6,4)$

   $m=\frac{6-2}{8-4}=1 \qquad m_{\perp}=-1$

   $y-4=-(x-6)$

6. $M=\left(\frac{-1+7}{2},\frac{1+(-5)}{2}\right)=(3,-2)$

   $m=\frac{1-(-5)}{-1-7}=-\frac{3}{4}$

   $m_{\perp}=\frac{4}{3}$

   $y+2=\frac{4}{3}(x-3)$

7. $M=\left(\frac{3+3}{2},\frac{-1+5}{2}\right)=(3,2)$

   Line segment is vertical, so its perpendicular bisector is horizontal, with equation $y = 2$.

8. $M=\left(\frac{0+6}{2},\frac{0+0}{2}\right)=(3,0)$

   Line segment is horizontal, so its perpendicular bisector is vertical, with equation $x = 3$.

Regents Questions

1) JUN '16 [12] Ans: 1
2) AUG '17 [24] Ans: 4

7.6 Directed Line Segments

Practice Problems

1. $k=\frac{5}{6}$ $P_x = A_x + \frac{5}{6}(B_x - A_x)$ $= 3 + \frac{5}{6}(9-3) = 8$ $P_y = A_y + \frac{5}{6}(B_y - A_y)$ $= 5 + \frac{5}{6}(17-5) = 15$ $P(8,15)$	2. $k=\frac{3}{5}$ $S_x = R_x + \frac{3}{5}(T_x - R_x)$ $= -2 + \frac{3}{5}(3+2) = 1$ $S_y = R_y + \frac{3}{5}(T_y - R_y)$ $= 2 + \frac{3}{5}(-8-2) = -4$ $S(1,-4)$
3. $k=\frac{3}{8}$ $P_x = L_x + \frac{3}{8}(M_x - L_x)$ $= -2 + \frac{3}{8}(6+2) = 1$ $P_y = L_y + \frac{3}{8}(M_y - L_y)$ $= 3 + \frac{3}{8}(-3-3) = \frac{3}{4}$ $P\left(1, \frac{3}{4}\right)$	4. $k=\frac{2}{5}$ $G_x = F_x + \frac{2}{5}(H_x - F_x)$ $= 1 + \frac{2}{5}(6-1) = 3$ $G_y = F_y + \frac{2}{5}(H_y - F_y)$ $= -3 + \frac{2}{5}(5+3) = \frac{1}{5}$ $G\left(3, \frac{1}{5}\right)$

Regents Questions

1) FALL '14 [14] Ans: 4
2) AUG '16 [18] Ans: 4
3) JAN '17 [20] Ans: 1
4) JUN '17 [15] Ans: 2
5) AUG '17 [17] Ans: 1
6) JAN '18 [6] Ans: 1
7) JUN '18 [14] Ans: 2
8) AUG '18 [15] Ans: 1
9) JAN '19 [15] Ans: 1
10) JUN '15 [27]

$P_x = -6 + \frac{2}{5}(4+6) = -2$

$P_y = -5 + \frac{2}{5}(0+5) = -3$

$P(-2,-3)$

11) AUG '15 [31]

$E_x = 1 + \frac{2}{5}(16-1) = 7$

$E_y = 4 + \frac{2}{5}(14-4) = 8$

$E(7,8)$

12) JAN '16 [27]

$J_x = -2 + \frac{2}{3}(4+2) = 2$

$J_y = 1 + \frac{2}{3}(7-1) = 5$

$J(2,5)$

13) JUN '16 [26]

$P_x = 4 + \frac{4}{9}(22-4) = 12$

$P_y = 2 + \frac{4}{9}(2-2) = 2$

$P(12,2)$

8. POLYGONS IN THE COORDINATE PLANE

8.1 Triangles in the Coordinate Plane

PRACTICE PROBLEMS

1.

$m_{\overline{AB}} = \frac{11-7}{5-2} = \frac{4}{3}$

$m_{\overline{BC}} = \frac{8-11}{9-5} = -\frac{3}{4}$

$\overline{AB} \perp \overline{BC}$, so $\angle B$ is a right angle.
Therefore, ABC is a right triangle.

2.

$AB = \sqrt{(5-2)^2 + (11-7)^2} = \sqrt{25} = 5$

$BC = \sqrt{(9-5)^2 + (8-11)^2} = \sqrt{25} = 5$

$AC = \sqrt{(9-2)^2 + (8-7)^2} = \sqrt{50} = 5\sqrt{2}$

Two sides are equal in length, so $\triangle ABC$ is an isosceles triangle.

3.

$DE = \sqrt{(5-4)^2 + (5+2)^2} = \sqrt{50} = 5\sqrt{2}$

$EF = \sqrt{(-1-5)^2 + (3-5)^2} = \sqrt{40} = 2\sqrt{10}$

$DF = \sqrt{(-1-4)^2 + (3+2)^2} = \sqrt{50} = 5\sqrt{2}$

Two sides are equal in length, so $\triangle DEF$ is an isosceles triangle.

4. $\overline{JK} \perp \overline{KL}$

$m_{\overline{JK}} = \frac{6-4}{6+2} = \frac{1}{4}$, so $m_{\overline{KL}} = -4$

$$\frac{-2-6}{x-6} = -4$$
$$-8 = -4(x-6)$$
$$-8 = -4x + 24$$
$$-32 = -4x$$
$$x = 8$$

5. Recognize that $\overline{PR}$ is horizontal (since $y = -2$ for both endpoints).

$PR = 8 - 4 = 4$

An altitude may be drawn from the opposite vertex, Q, to point $S(-6,-2)$.

QS = $4 - (-2) = 6$

Therefore, the area is $\frac{1}{2}(4)(6) = 12$ square units.

6.

$AB = \sqrt{(5-2)^2 + (1-2)^2} = \sqrt{10}$

$DE = \sqrt{(4-1)^2 + (-5+4)^2} = \sqrt{10}$

$BC = \sqrt{(4-5)^2 + (5-1)^2} = \sqrt{17}$

$EF = \sqrt{(3-4)^2 + (-1+5)^2} = \sqrt{17}$

$AC = \sqrt{(4-2)^2 + (5-2)^2} = \sqrt{13}$

$DF = \sqrt{(3-1)^2 + (-1+4)^2} = \sqrt{13}$

$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, and $\overline{AC} \cong \overline{DF}$, so $\triangle ABC \cong \triangle DEF$ by SSS.

7. $m_{\overline{RS}} = \frac{-1-7}{3+1} = -\frac{8}{4} = -2$

$m_{\overline{ST}} = \frac{2+1}{9-3} = \frac{3}{6} = \frac{1}{2}$

The slopes are opposite reciprocals, so they are perpendicular lines forming a right angle at *S*.

Since $\angle S$ is a right angle, $\triangle RST$ is a right triangle.

8. To prove that $\triangle JEN$ is a right triangle, prove that its legs are perpendicular by showing their slopes are opposite reciprocals:

$m_{\overline{JE}} = \frac{-3-1}{-2+4} = -2$

$m_{\overline{EN}} = \frac{-1+3}{2+2} = \frac{1}{2}$

To prove that $\triangle JEN$ is an isosceles triangle, prove that it legs are congruent by using the distance formula:

$JE = \sqrt{(-2+4)^2 + (-3-1)^2} = \sqrt{20}$

$EN = \sqrt{(2+2)^2 + (-1+3)^2} = \sqrt{20}$

Regents Questions

1) JAN '16 [18] Ans: 1

2) AUG '16 [15] Ans: 3

3) AUG '15 [33]

$m_{\overline{BC}} = -\frac{3}{2}$ $m_{\perp} = \frac{2}{3}$

The right angle may be at *B* or *C*.

Right angle at *B* means:

$-1 = \frac{2}{3}(-3) + b$ $3 = \frac{2}{3}x + 1$

$b = 1$ $x = 3$

Right angle at *C* means:

$-4 = \frac{2}{3}(-1) + b$

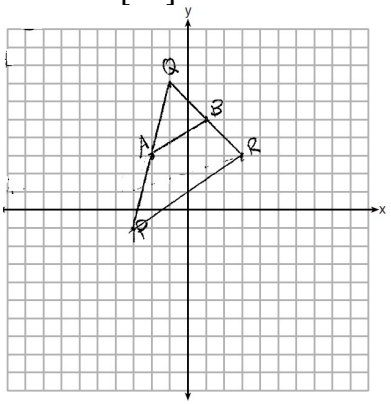
$b = -\frac{10}{3}$

$3 = \frac{2}{3}x - \frac{10}{3}$

$x = \frac{19}{2} = 9.5$

Either answer, $x = 3$ or $x = 9.5$, is correct.

4) AUG '17 [32]



$A(-2,3)$ and $B(1,5)$

$m_{\overline{AB}} = \frac{5-3}{1-(-2)} = \frac{2}{3}$

$m_{\overline{PR}} = \frac{3-(-1)}{3-(-3)} = \frac{4}{6} = \frac{2}{3}$

$\overline{AB} \parallel \overline{PR}$ because same slopes.

$AB = \sqrt{3^2 + 2^2} = \sqrt{13}$

$PR = \sqrt{6^2 + 4^2} = \sqrt{52} = 2\sqrt{13}$

Therefore, $AB = \frac{1}{2}PR$.

5) JUN '18 [32]

$AB = \sqrt{5^2 + 1^2} = \sqrt{26}$

$BC = \sqrt{4^2 + 4^2} = \sqrt{32}$

$AC = \sqrt{1^2 + 5^2} = \sqrt{26}$

$\triangle ABC$ has two congruent sides, not three, so it is isosceles and not equilateral.

6) JAN '19 [32]

$AC = \sqrt{3^2 + (-5)^2} = \sqrt{34}$

$BC = \sqrt{(-5)^2 + (-3)^2} = \sqrt{34}$

Isosceles because AC = BC

$m_{\overline{AC}} = \frac{-1-4}{1-(-2)} = -\frac{5}{3}$

$m_{\overline{BC}} = \frac{-1-2}{1-6} = \frac{3}{5}$

$\overline{AC} \perp \overline{BC}$ because their slopes are opposite reciprocals, so $\angle C$ is a right angle

8.2 Quadrilaterals in the Coordinate Plane

Practice Problems

1.

$m_{\overline{AB}} = \frac{-8-0}{-1+5} = -2 \qquad m_{\overline{CD}} = \frac{4+4}{3-7} = -2$

$m_{\overline{BC}} = \frac{-4+8}{7+1} = \frac{1}{2} \qquad m_{\overline{AD}} = \frac{4-0}{3+5} = \frac{1}{2}$

$\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{CD}$, $\overline{CD} \perp \overline{AD}$, and $\overline{AD} \perp \overline{AB}$, so all 4 angles are right angles.
Therefore, *ABCD* is a rectangle.

2.

$AB = \sqrt{(1+6)^2 + (0+3)^2} = \sqrt{58}$

$BC = \sqrt{(4-1)^2 + (7-0)^2} = \sqrt{58}$

$CD = \sqrt{(-3-4)^2 + (4-7)^2} = \sqrt{58}$

$AD = \sqrt{(-3+6)^2 + (4+3)^2} = \sqrt{58}$

$m_{\overline{AB}} = \frac{0+3}{1+6} = \frac{3}{7} \qquad m_{\overline{BC}} = \frac{7-0}{4-1} = \frac{7}{3}$

4 congruent sides but not 4 right angles = Rhombus

3.

$AB = \sqrt{(2+5)^2 + (0+6)^2} = \sqrt{85}$

$BC = \sqrt{(11-2)^2 + (9-0)^2} = \sqrt{162}$

$CD = \sqrt{(4-11)^2 + (3-9)^2} = \sqrt{85}$

$AD = \sqrt{(4+5)^2 + (3+6)^2} = \sqrt{162}$

$m_{\overline{AB}} = \frac{0+6}{2+5} = \frac{6}{7} \qquad m_{\overline{BC}} = \frac{9-0}{11-2} = 1$

2 pairs of opposite sides congruent but not 4 right angles = Parallelogram

4.

$AB = \sqrt{(5-1)^2 + (2-1)^2} = \sqrt{17}$

$BC = \sqrt{(6-5)^2 + (-2-2)^2} = \sqrt{17}$

$CD = \sqrt{(2-6)^2 + (-3+2)^2} = \sqrt{17}$

$AD = \sqrt{(2-1)^2 + (-3-1)^2} = \sqrt{17}$

$m_{\overline{AB}} = \frac{2-1}{5-1} = \frac{1}{4} \qquad m_{\overline{BC}} = \frac{-2-2}{6-5} = -4$

$\overline{AB} \perp \overline{BC}$, so $\angle B$ is a right angle.
4 congruent sides and a right angle = Square

5. $m_{\overline{AB}} = \frac{4-1}{4-2} = \frac{3}{2}$

Since $\overline{AB} \perp \overline{BC}$, $m_{\overline{BC}} = -\frac{2}{3}$.

6. $PR = \sqrt{(7+4)^2 + (-5-0)^2} = \sqrt{146}$

$QS = \sqrt{(-1-4)^2 + (-8-3)^2} = \sqrt{146}$

Diagonals are congruent.

$m_{\overline{PR}} = \frac{-5-0}{7+4} = -\frac{5}{11}$

$m_{\overline{QS}} = \frac{-8-3}{-1-4} = \frac{11}{5}$

Diagonals are perpendicular.

Midpoint of $\overline{PR}$ is

$\left(\frac{-4+7}{2}, \frac{0-5}{2}\right) = \left(\frac{3}{2}, -\frac{5}{2}\right)$

and midpoint of $\overline{QS}$ is

$\left(\frac{4-1}{2}, \frac{3-8}{2}\right) = \left(\frac{3}{2}, -\frac{5}{2}\right)$

Diagonals bisect each other.

7. Both pairs of opposite sides of a parallelogram are parallel. Parallel lines have the same slope.

The slope of side $\overline{BC}$ is 3. For side $\overline{AD}$ to have a slope of 3, the coordinates of point *D* must be (1,3).

$m_{\overline{AB}} = \frac{2-0}{5-0} = \frac{2}{5} \quad m_{\overline{CD}} = \frac{3-5}{1-6} = \frac{2}{5}$

$m_{\overline{AD}} = \frac{3-0}{1-0} = 3 \quad m_{\overline{BC}} = \frac{5-2}{6-5} = 3$

8. To prove that *ABCD* is a rhombus, show that all sides are congruent using the distance formula:

$AB = \sqrt{(8+1)^2 + (2+5)^2} = \sqrt{130}$

$BC = \sqrt{(11-8)^2 + (13-2)^2} = \sqrt{130}$

$CD = \sqrt{(2-11)^2 + (6-13)^2} = \sqrt{130}$

$AD = \sqrt{(2+1)^2 + (6+5)^2} = \sqrt{130}$

9. To prove that *ABCD* is a parallelogram, show that both pairs of opposite sides of the parallelogram are parallel by showing the opposite sides have the same slope:

$m_{\overline{AB}} = \frac{5-2}{6+2} = \frac{3}{8} \quad m_{\overline{CD}} = \frac{-3-0}{-4-4} = \frac{3}{8}$

$m_{\overline{AD}} = \frac{-3-2}{-4+2} = \frac{5}{2} \quad m_{\overline{BC}} = \frac{0-5}{4-6} = \frac{5}{2}$

Because $\frac{3}{8}$ and $\frac{5}{2}$ are not opposite reciprocals, the consecutive sides of *ABCD* are not perpendicular, and *ABCD* is not a rectangle.

10. The length of each side of quadrilateral is 5. Since each side is congruent, quadrilateral *MATH* is a rhombus.

The slope of $\overline{MH}$ is 0 and the slope of $\overline{HT}$ is $-\frac{4}{3}$. Since the slopes are not opposite reciprocals, the sides are not perpendicular and do not form rights angles.

Since adjacent sides are not perpendicular, quadrilateral *MATH* is not a square.

11. $m_{\overline{AB}} = \frac{6-6}{6+5} = 0 \quad m_{\overline{CD}} = \frac{-3+3}{-3-8} = 0$

$m_{\overline{AD}} = \frac{-3-6}{-3+5} = -\frac{9}{2} \quad m_{\overline{BC}} = \frac{-3-6}{8-6} = -\frac{9}{2}$

$\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$ because they have equal slopes. *ABCD* is a parallelogram because opposite side are parallel.

AB = 11 and $BC = \sqrt{85}$. *ABCD* is not a rhombus because $AB \neq BC$.

$\overline{AB}$ and $\overline{BC}$ are not perpendicular because their slopes are not opposite reciprocals. Therefore, *ABCD* is not a rectangle because $\angle B$ is not a right angle.

12. Use the midpoint formula to find *M*(–5,5), *N*(0,3), *P*(2,–4), *Q*(–3,–2).

Use the slope formula to find

$m_{\overline{MN}} = -\frac{2}{5} \qquad m_{\overline{PQ}} = -\frac{2}{5}$

$m_{\overline{MQ}} = -\frac{7}{5} \qquad m_{\overline{NP}} = -\frac{7}{5}$

Since both opposite sides have equal slopes and are parallel, *MNPQ* is a parallelogram.

Use the distance formula to find $MN = \sqrt{29}$ and $NP = \sqrt{53}$.

$MN \neq NP$, so *MNPQ* is not a rhombus since not all sides are congruent.

REGENTS QUESTIONS

1) AUG '15 [22] Ans: 4
2) AUG '16 [14] Ans: 1
3) JAN '17 [19] Ans: 3
4) FALL '14 [11]

Midpoint of $\overline{MT}$ is $\left(\frac{0+4}{2},\frac{-1+6}{2}\right)=\left(2,\frac{5}{2}\right)$.

$m_{\overline{MT}}=\frac{6+1}{4-0}=\frac{7}{4}$. In a rhombus, diagonals are perpendicular, so $m_{\overline{AH}}=-\frac{4}{7}$.

$y-\frac{5}{2}=-\frac{4}{7}(x-2)$

The diagonals, $\overline{MT}$ and $\overline{AH}$, of rhombus *MATH* are perpendicular bisectors of each other.

5) JUN '15 [36]

$m_{\overline{TS}}=-\frac{5}{3}$ $m_{\overline{SR}}=\frac{3}{5}$

Since the slopes of $\overline{TS}$ and $\overline{SR}$ are opposite reciprocals, they are perpendicular and form a right angle. $\triangle RST$ is a right triangle because $\angle S$ is a right angle.

$P(0,9)$

$m_{\overline{RP}}=-\frac{5}{3}$ $m_{\overline{PT}}=\frac{3}{5}$

Since the slopes of all four adjacent sides are opposite reciprocals, they are perpendicular and form right angles. Quadrilateral *RSTP* is a rectangle because it has four right angles.

6) JAN '17 [31]

(1,3) and (–3,–3)

7) JUN '17 [35]

$PQ=\sqrt{5^2+5^2}=\sqrt{50}$

$QR=\sqrt{(-7)^2+1^2}=\sqrt{50}$

$RS=\sqrt{(-5)^2+(-5)^2}=\sqrt{50}$

$PS=\sqrt{(-7)^2+1^2}=\sqrt{50}$

PQRS is a rhombus because all sides are congruent. The slope of $\overline{PQ}$ is 1 and the slope of $\overline{QR}$ is –7. Because the slopes of adjacent sides are not opposite reciprocals, they are not perpendicular and do not form a right angle. Therefore *PQRS* is not a square.

8) JAN '18 [35]

$PA=\sqrt{(-4+1)^2+(5+6)^2}=\sqrt{130}$

$AT=\sqrt{(5+4)^2+(-2-5)^2}=\sqrt{130}$

$PA=AT$, so $\triangle PAT$ is isosceles.

$R(2,9)$.

$m_{\overline{PA}}=-\frac{11}{3}$ and $m_{\overline{RT}}=-\frac{11}{3}$, so $\overline{PA}\parallel\overline{RT}$.

$RT=\sqrt{(5-2)^2+(-2-9)^2}=\sqrt{130}$, so $\overline{PA}\cong\overline{RT}$.

PART is a parallelogram because it has a pair of opposite sides that are both congruent and parallel.

9) AUG '18 [35]

$m_{\overline{MH}}=\frac{6}{10}=\frac{3}{5}$ and $m_{\overline{AT}}=\frac{6}{10}=\frac{3}{5}$

$m_{\overline{MA}}=-\frac{5}{3}$ and $m_{\overline{HT}}=-\frac{5}{3}$

$\overline{MH}\parallel\overline{AT}$ and $\overline{MA}\parallel\overline{HT}$, so *MATH* is a parallelogram since both sides of opposite sides are parallel.

Since their slopes are negative reciprocals, $\overline{MA}\perp\overline{AT}$ and $\angle A$ is a right angle. *MATH* is a rectangle because it is a parallelogram with a right angle.

8.3 Perimeter and Area using Coordinates

PRACTICE PROBLEMS

1.

$AB = \sqrt{(4-1)^2 + (1-4)^2} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(-3-4)^2 + (0-1)^2} = \sqrt{50} = 5\sqrt{2}$

$AC = \sqrt{(-3-1)^2 + (0-4)^2} = \sqrt{32} = 4\sqrt{2}$

Perimeter = $3\sqrt{2} + 5\sqrt{2} + 4\sqrt{2} = 12\sqrt{2}$

2.

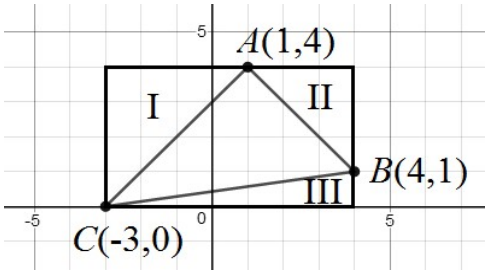
$AB = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$

$BC = \sqrt{8^2 + (-6)^2} = \sqrt{100} = 10$

$AC = \sqrt{11^2 + (-2)^2} = \sqrt{125} = 5\sqrt{5}$

Perimeter = $5 + 10 + 5\sqrt{5} = 15 + 5\sqrt{5}$

3. *Method 1: enclose in rectangle*



Area of rectangle = 7×4 = 28

Area of I = $\frac{1}{2}(4)(4) = 8$

Area of II = $\frac{1}{2}(3)(3) = 4.5$

Area of III = $\frac{1}{2}(7)(1) = 3.5$

Area of $\triangle ABC$ = 28 – (8 + 4.5 + 3.5) = 12 square units

Method 2: area of right triangle

$m_{\overline{CA}} = \frac{4}{4} = 1 \quad m_{\overline{AB}} = -\frac{3}{3} = -1$

Slopes are opposite reciprocals, so $\overline{CA} \perp \overline{AB}$ and $\angle A$ is a right angle.

In problem 1, $AC = 4\sqrt{2}$ and $AB = 3\sqrt{2}$.

Area of right triangle $ABC = \frac{1}{2}(AC)(AB)$

$= \frac{1}{2}(4\sqrt{2})(3\sqrt{2}) = 12$ square units.

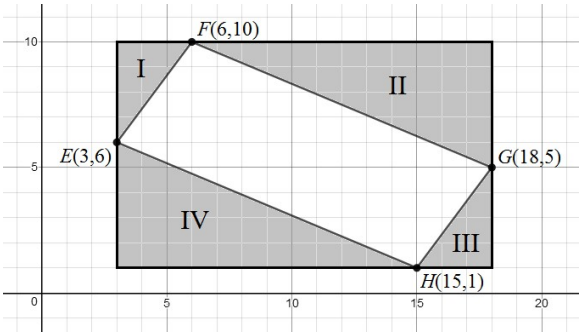
4.

$EF = \sqrt{(6-3)^2 + (10-6)^2} = \sqrt{25} = 5$

$FG = \sqrt{(18-6)^2 + (5-10)^2} = \sqrt{169} = 13$

Since *EFGH* is a parallelogram, *GH = EF* and *EH = FG*.

Perimeter of EFGH = 2×5 + 2×13 = 36



Area of rectangle = 15×9 = 135

Area of I = Area of III = $\frac{1}{2}(3)(4) = 6$

Area of II = Area of IV = $\frac{1}{2}(5)(12) = 30$

Area of $\square$EFGH = 135 – (2×6 + 2×30) = 63 square units

5.

vertex	*x*	*y*	*upper*	*lower*	*difference*
K	–7	–7	–14	35	–49
L	–5	2	–30	6	–36
M	3	6	–9	6	–15
N	1	–3	–7	21	–28
K	–7	–7			

Area = $\frac{|(-49)+(-36)+(-15)+(-28)|}{2} = \frac{|128|}{2} = 64$ square units.

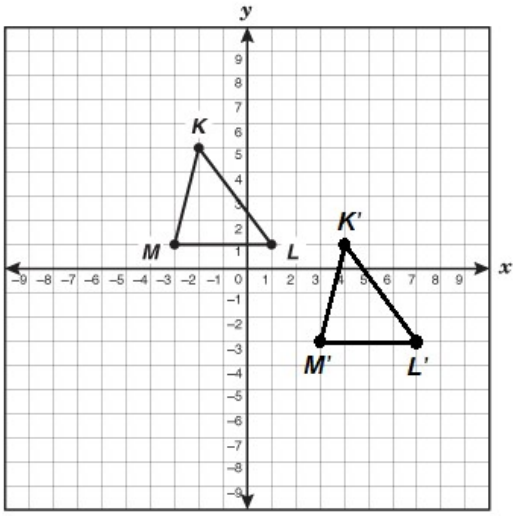
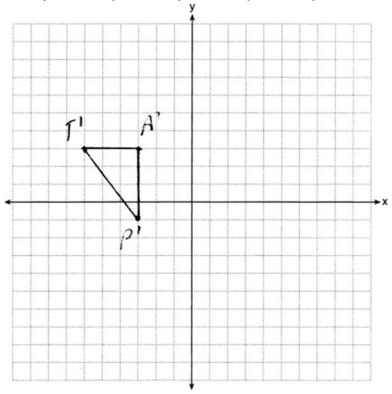
REGENTS QUESTIONS

1) JUN '16 [22] Ans: 3
2) JUN '17 [2] Ans: 3
3) AUG '17 [3] Ans: 3
4) JUN '18 [15] Ans: 1
5) AUG '18 [8] Ans: 4
6) JAN '19 [21] Ans: 4

9. RIGID MOTIONS

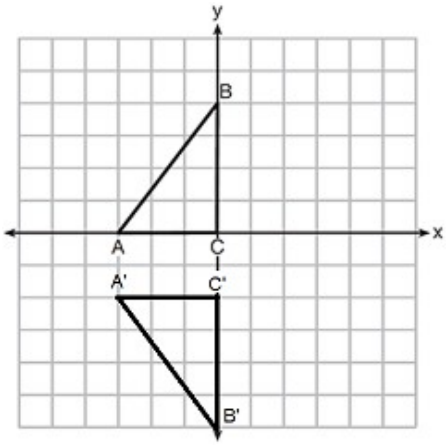
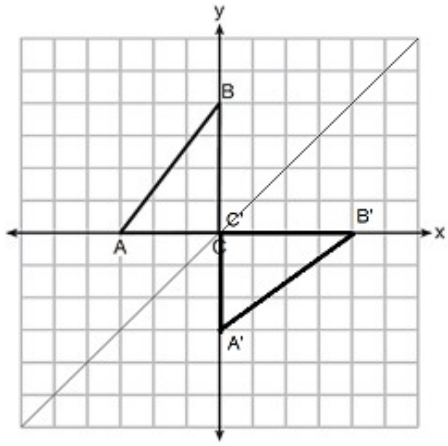
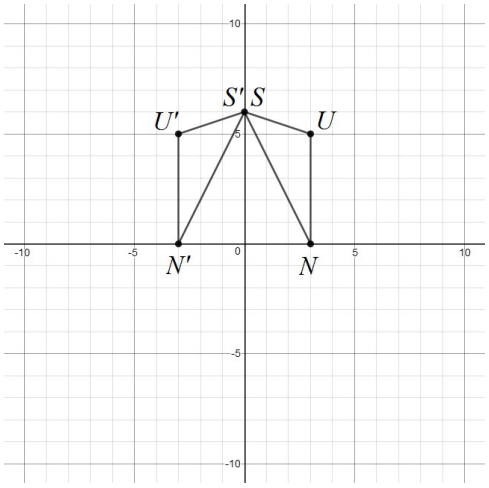
9.1 Translations

PRACTICE PROBLEMS

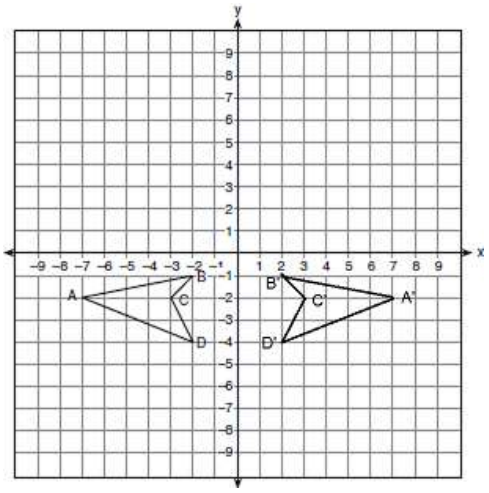
1. $(2-6,4+1)=(-4,5)$	2. $(-2,-2)$
3. $(x+3, y-7)$	4. $(2,-8)$
5. $(-6,6)$	6. $(x+4, y+4)$
7. $(0,-9)$	8. $T_{2,2}$ of Q is $Q'(6,-4)$
9. $R(-5,-5) \rightarrow R'(3,0)$ is $T_{8,5}$ $U'(3,6)$	10. $A(1,3) \rightarrow A'(4,4)$ is $T_{3,1}$ $C'(7,1)$
11. $(-5,5)$	12. $(0,1)$
13. $B(-6,4)$ and $D'(-5,-4)$	
14. 	
15. $T'(-6,3)$, $A'(-3,3)$, $P'(-3,-1)$ 	

9.2 Line Reflections

Practice Problems

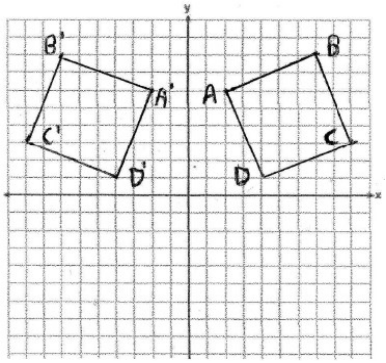
1. (2,3)	2. $P'(-4,1)$
3. (2,5)	4. (−3,4)
5. $M'(2,8)$	6. (−4,3)
7. (−2,5)	8. $A'(0,-2)$ and $B'(4,-6)$
9. 	10. 
11. $X'(5,1)$, $Y'(4,4)$, $Z'(7,4)$	12.  $S'(0,6)$, $U'(-3,5)$, $N'(-3,0)$

13.



A'(7,–2), B'(2,–1), C'(3,–2), D'(2,–4)

14.

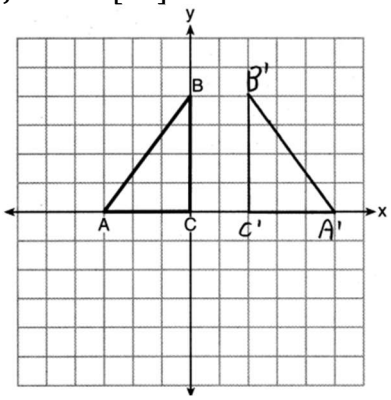


$AB = \sqrt{(7-2)^2 + (8-6)^2} = \sqrt{29}$

Area = $\left(\sqrt{29}\right)^2 = 29$

REGENTS QUESTIONS

1) JAN '16 [25]



9.3 Rotations

Practice Problems

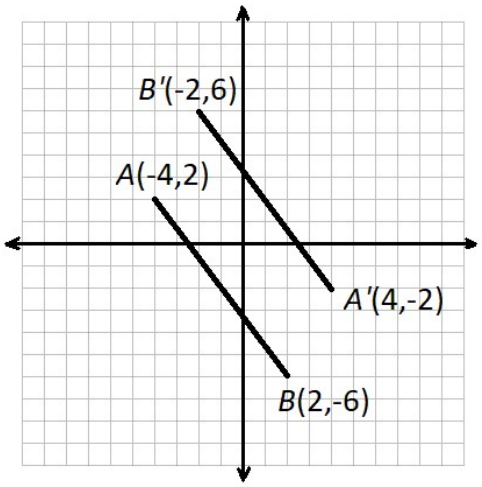
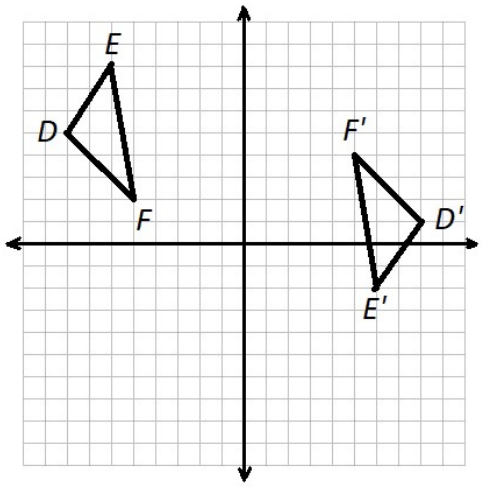
1. (3)	2. (4)
3. (2)	
4. $R_{90°}:(x,y)\rightarrow(-y,x)$ $(2,4)\rightarrow(-4,2)$	
5. A clockwise rotation of 90° is equivalent to a counterclockwise rotation of 270°. $R_{270°}:(x,y)\rightarrow(y,-x)$ $(-2,5)\rightarrow(5,2)$	6. A clockwise rotation of 180° is equivalent to a counterclockwise rotation of 180°. $R_{180°}:(x,y)\rightarrow(-x,-y)$ $(-2,1)\rightarrow(2,-1)$
7. $R_{180°}:(x,y)\rightarrow(-x,-y)$ $A'(0,4)$, $B'(-4,2)$, $C'(-5,4)$, $D'(-1,6)$	8. $R_{90°}:(x,y)\rightarrow(-y,x)$ $A'(-2,1)$, $B'(-3,-4)$, $C'(5,-3)$

Regents Questions

1) FALL '14 [2] Ans: 4
2) JAN '16 [11] Ans: 4
3) AUG '16 [5] Ans: 1
4) AUG '16 [29]
$m\angle P = m\angle L = 47°$ because rotations preserve angles.
$m\angle M = 180-(47+57) = 76$
because the angles of a triangle add to 180°.

9.4 Point Reflections

Practice Problems

1. $K'(-4,7)$	2. $(3,1)$
3. $R'(-2,3), S'(-5,-1)$	4. $(2\cdot 1-5, 2\cdot(-1)-3) = N'(-3,-5)$
5. $2\times 6 = 12$ units	6. Use the midpoint formula: $P\left(\frac{3+7}{2}, \frac{6-2}{2}\right) = P(5,2)$
7. 	8. $D'(8,1)$, $E'(6,-2)$, $F'(5,4)$ 

9.5 Map a Polygon onto Itself

Practice Problems

1. (3)	2. 180°
3. $\frac{360}{5} = 72°$	4. $\frac{360}{16} = 22.5°$

Regents Questions

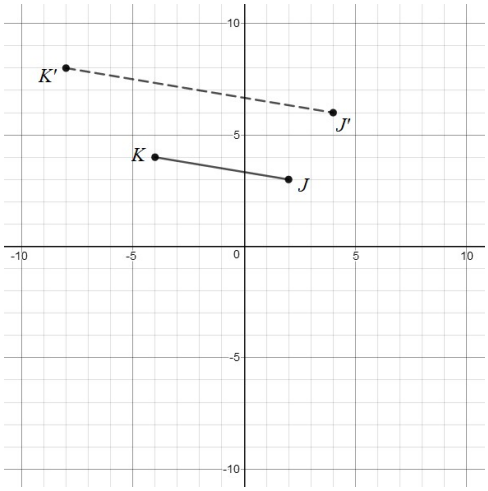
1) FALL '14 [15] Ans: 2
2) JUN '15 [10] Ans: 1
3) AUG '15 [5] Ans: 1
4) JAN '17 [17] Ans: 4
5) JUN '17 [7] Ans: 1
6) AUG '17 [6] Ans: 3
7) AUG '17 [22] Ans: 4
8) JAN '18 [15] Ans: 3
9) JUN '18 [19] Ans: 3
10) AUG '18 [17] Ans: 3
11) JAN '19 [4] Ans: 3
12) AUG '16 [27]

$\frac{360}{6} = 60$

10. DILATIONS

10.1 Dilations of Line Segments

Practice Problems

1. (1)	
2. (15,–10)	3. (4,12)
4. $\left(\frac{3}{2},-1\right)$	5. (2,5)
6. (12,0)	7. 

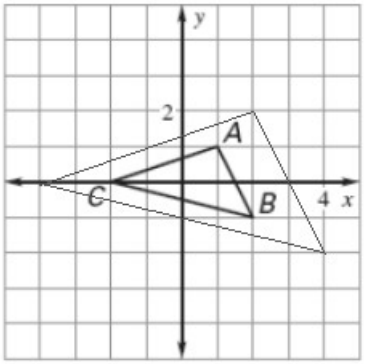
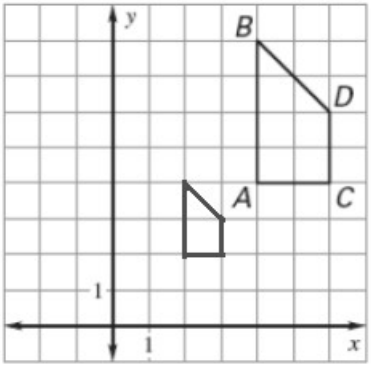
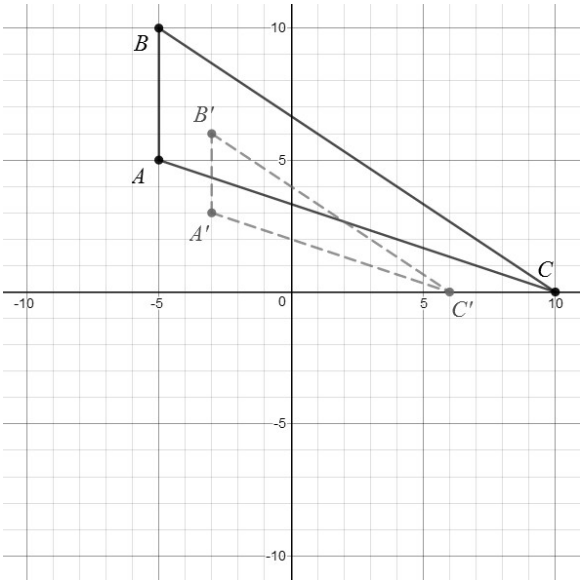
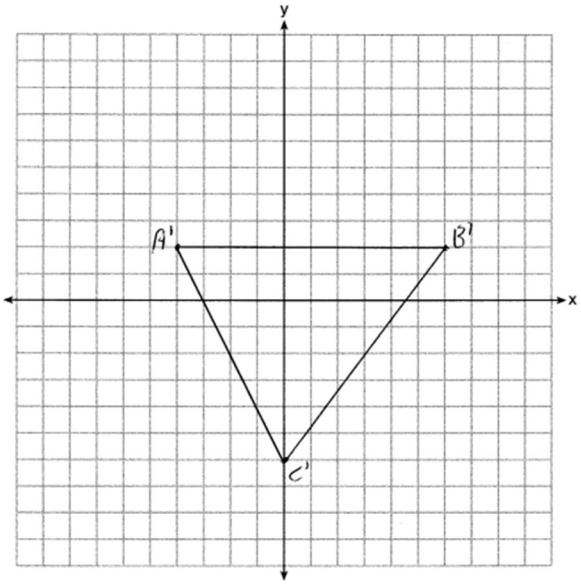
Regents Questions

1) JUN '15 [18] Ans: 1
2) JAN '16 [10] Ans: 2
3) JUN '16 [2] Ans: 4
4) AUG '16 [21] Ans: 4
5) JAN '17 [13] Ans: 1
6) JUN '17 [6] Ans: 3
7) AUG '17 [10] Ans: 1
8) JAN '19 [1] Ans: 4
9) AUG '17 [29]

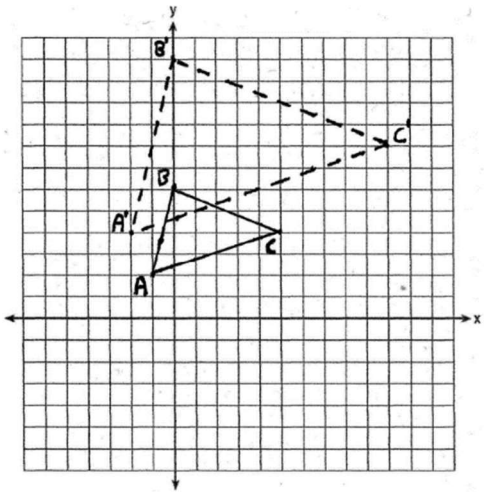
$$A'B' = \frac{1}{2}AB$$
$$= \frac{1}{2}\sqrt{(5-2)^2+(-1-3)^2}$$
$$= \frac{1}{2}\sqrt{25} = 2.5$$

10.2 Dilations of Polygons

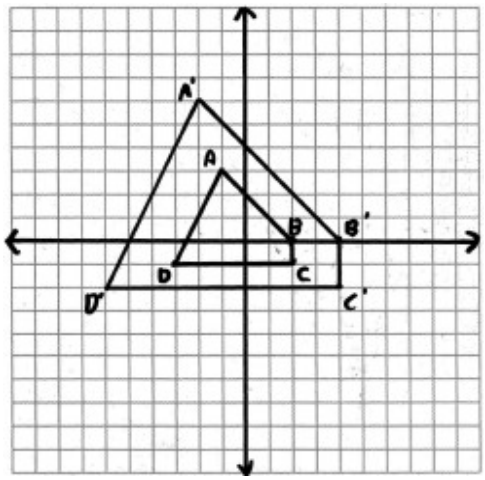
Practice Problems

1. (3) $\text{m}\angle B = \text{m}\angle B'$	2. $A'(2,2), B'(3,0), C'(1,-1)$
3. 	4. 
5. 	6. 

7.



8.



$A'(-2,6)$, $B'(4,0)$, $C'(4,-2)$, $D'(-6,-2)$

9. (a) $A'(-1,1)$, $B'(4,-2)$, $C'(3,-5)$, $D'(-2,-2)$

(b)

$$m_{\overline{A'B'}} = \frac{-2-1}{4+1} = -\frac{3}{5}$$

$$m_{\overline{C'D'}} = \frac{-2+5}{-2-3} = -\frac{3}{5}$$

$$m_{\overline{A'D'}} = \frac{-2-1}{-2+1} = 3$$

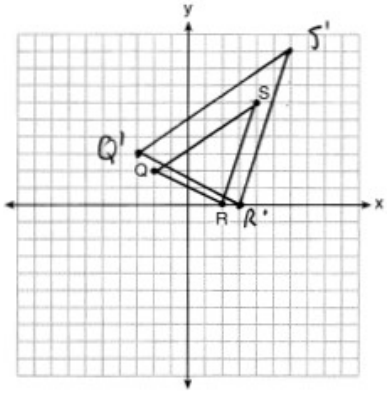
$$m_{\overline{B'C'}} = \frac{-5+2}{3-4} = 3$$

$\overline{A'B'} \parallel \overline{C'D'}$ and $\overline{A'D'} \parallel \overline{B'C'}$

$A'B'C'D'$ is a parallelogram because both pairs of opposite sides are parallel.

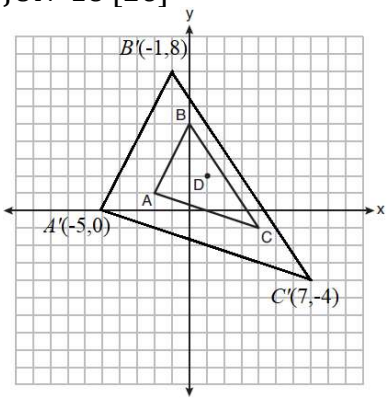
Regents Questions

1) JUN '15 [16] Ans: 2
2) AUG '15 [6] Ans: 4
3) AUG '15 [20] Ans: 1
4) AUG '15 [23] Ans: 1
5) JAN '18 [11] Ans: 1
6) JUN '18 [5] Ans: 4
7) AUG '18 [23] Ans: 3
8) JAN '17 [32]



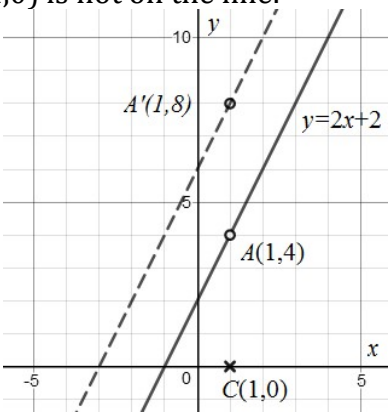
A dilation preserves slope, so the slopes of $\overline{QR}$ and $\overline{Q'R'}$ are equal. Therefore, $\overline{Q'R'} \parallel \overline{QR}$.

9) JUN '18 [26]



10.3 Dilations of Lines

Practice Problems

1. $y = 3x - 20$	2. $2x + 3y = 4 \rightarrow y = -\frac{2}{3}x + \frac{4}{3}$ Equation of its image is $y = -\frac{2}{3}x + 4$
3. $C(1,-1)$ is a point on the line: $(-1) = 3(1) - 4$ Therefore, the equation of the image is the same as the pre-image, $y = 3x - 4$.	4. $C(1,0)$ is not on the line.  Find the point on the line where $x = 1$: $y = 2(1) + 2 = 4$, so $A(1,4)$. $CA = 4$, so $CA' = 2(4) = 8$ and $A'(1,8)$ $8 = 2(1) + b \rightarrow b = 6$ Equation of image is $y = 2x + 6$.

Regents Questions

1) FALL '14 [3] Ans: 2
2) FALL '14 [16] Ans: 2
3) JUN '15 [22] Ans: 1
4) AUG '15 [24] Ans: 4
5) JAN '18 [14] Ans: 1
6) JUN '18 [24] Ans: 2
7) JAN '19 [24] Ans: 4
8) JAN '16 [31]
$\ell: y = 3x - 4$
$m: y = 3x - 8$
9) JUN '17 [31]
The center of dilation is on the original line, so the line does not change. Line p is $3x + 4y = 20$.
10) AUG '18 [30]
No, the line passes through the center of dilation, so the dilated line is not distinct. To show the given lines are distinct,
$4x + 3y = 24$
$3y = -4x + 24$
$y = -\frac{4}{3}x + 8$
This has a different y-intercept than $y = -\frac{4}{3}x + 16$.

11. TRANSFORMATION PROOFS

11.1 Properties of Transformations

Practice Problems

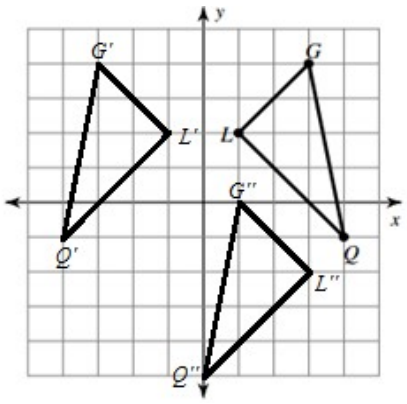
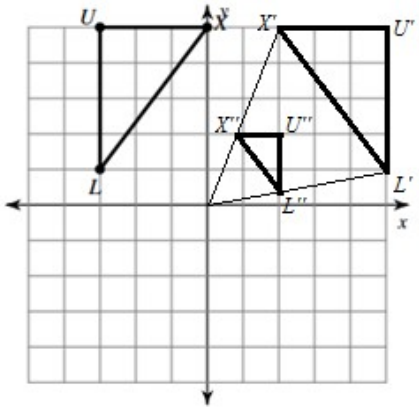
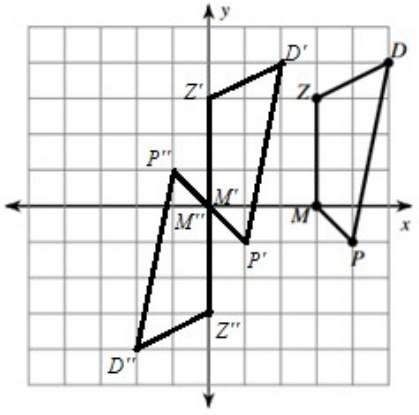
1. (a) reflection (b) dilation (c) translation (d) rotation (e) dilation	
2. (3) rotation	3. (2) opposite orientation; reflection
4. (4) $r_{y=x}$	5. (3) $r_{y\text{-}axis}$
6. (3) orientation	7. (4) D_2
8. (1) dilation	9. (3) dilation by a scale factor of $\frac{1}{2}$

Regents Questions

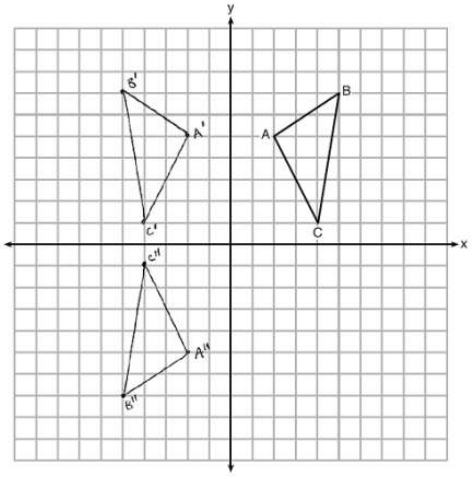
1)	AUG '15 [13]	Ans: 2	5)	JAN '18 [4]	Ans: 4
2)	JAN '16 [5]	Ans: 3	6)	JAN '18 [8]	Ans: 4
3)	JUN '16 [4]	Ans: 1	7)	JUN '18 [1]	Ans: 1
4)	AUG '16 [2]	Ans: 2	8)	JAN '19 [2]	Ans: 4

11.2 Sequences of Transformations

Practice Problems

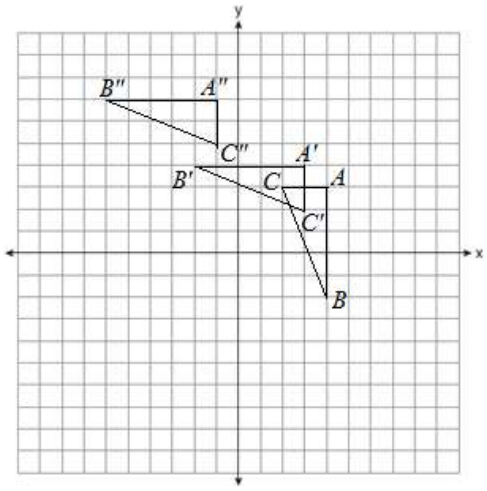
1. (4) translation followed by a reflection	
2. Rotation R_{90° with center at origin, followed by reflection r_{x-axis}	3. Rotation R_{270° around point A, followed by translation $T_{5,-2}$
4. Reflection $r_{y=\frac{1}{2}}$ followed by rotation R_{270° around the point (1,–3)	5. 
6. 	7. 
8. (3) (8,12)	

9.



$R_{180°}$

10.

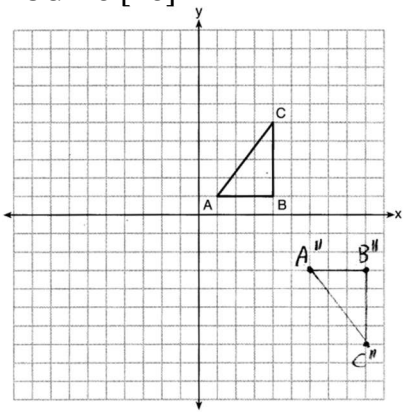


$A'(3,4)$, $B'(-2,4)$, $C'(3,2)$
$A''(-1,7)$, $B''(-6,7)$, $C''(-1,5)$

11. If $\overline{AB}$ is reflected first, the coordinates are $A'(2,-6)$ and $B'(4,-2)$.
When the reflection is dilated, the coordinates are $A''(1,-3)$ and $B''(2,-1)$.
If $\overline{AB}$ is dilated first, the coordinates are $A'(1,3)$ and $B'(2,1)$.
When the dilation is reflected, the coordinates are $A''(1,-3)$ and $B''(2,-1)$.
The images are the same.

Regents Questions

1) JUN '15 [4] Ans: 4
2) AUG '15 [7] Ans: 1
3) JAN '16 [8] Ans: 1
4) JUN '16 [8] Ans: 4
5) JAN '17 [10] Ans: 3
6) JUN '17 [1] Ans: 2
7) JUN '18 [3] Ans: 4
8) AUG '18 [4] Ans: 1
9) JAN '19 [3] Ans: 3
10) JUN '16 [25]
translation 6 units right and reflection over *x*-axis
11) AUG '16 [26]



12) JAN '17 [26]
$T_{0,-2}$ and r_{y-axis}
13) JUN '17 [30]
Rotate $\triangle ABC$ about point *C* until $\overline{DF} \parallel \overline{AC}$. Translate $\triangle ABC$ along $\overline{CF}$ so that *C* maps onto *F*.
14) AUG '17 [27]
$R_{180°}$ about $\left(-\frac{1}{2},\frac{1}{2}\right)$
15) JUN '18 [27]
Reflection over the *y*-axis followed by a translation up 5.
16) AUG '18 [28]
rotation 180º about (0,–1); or rotation 180º about the origin, translation 2 units down; or reflection over x-axis, translation 2 units down, reflection over y-axis
17) JAN '19 [28]
reflection over line: $r_{(y = x + 4)}$; or rotation $R_{90°}$ around (–5,–1) followed by reflection $r_{(x = -1)}$; or rotation $R_{90°}$ around (–5,2) followed by $T_{3,-1}$ and r_{x-axi}

11.3 Transformations and Congruence

Practice Problems

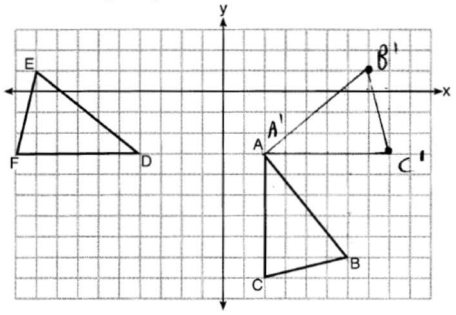
1. Translation so that A maps onto D.	2. Reflection over $\overline{AC}$.
3. Rotation of 180° around point J.	4. Reflection over the bisector of $\angle HLK$. *(Note that $\triangle HLK$ is isosceles.)*
5. Reflection over the vertical line passing through P, followed by the translation $T_{2,-4}$ (or vice versa). No, it is not possible because the triangles have opposite orientations and both translations and rotations preserve orientations.	

Regents Questions

1) JUN '15 [24] Ans: 3
2) JUN '16 [16] Ans: 3
3) JAN '17 [6] Ans: 4
4) AUG '17 [2] Ans: 4
5) JAN '18 [1] Ans: 1
6) FALL '14 [4]
Translate $\triangle ABC$ such that point C maps onto point F, then reflect over $\overline{DF}$.
or
Reflect $\triangle ABC$ over the perpendicular bisector of $\overline{EB}$.
7) FALL '14 [21]
a) $\overline{LA} \cong \overline{DN}, \overline{CA} \cong \overline{CN}$, and $\overline{DAC} \perp \overline{LCN}$ (Given).
$\angle LCA$ and $\angle DCN$ are right angles (Definition of perpendicular lines).
$\triangle LAC$ and $\triangle DNC$ are right triangles (Definition of a right triangle). $\triangle LAC \cong \triangle DNC$ (HL).
b) Rotate $\triangle LAC$ counterclockwise 90º about point C such that point L maps onto point D.
8) JUN '15 [30]
Reflections are rigid motions that preserve congruency.
9) JUN '15 [33]
Quadrilateral $ABCD$ is a parallelogram with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at E (Given).
$\overline{AD} \cong \overline{BC}$ (Opposite sides of a parallelogram are congruent).
$\angle AED \cong \angle CEB$ (Vertical angles are congruent).
$\overline{BC} \parallel \overline{DA}$ (Definition of parallelogram).
$\angle DBC \cong \angle BDA$ (Alternate interior angles are congruent).
$\triangle AED \cong \triangle CEB$ (AAS).
180° rotation of $\triangle AED$ around point E.
10) AUG '15 [30]
The transformation is a rotation of 180° around the origin. Rotations are rigid motions that preserve congruency.
11) AUG '15 [34]
Translations preserve distance, so if point D is mapped onto point A, point F would map onto point C.
$\triangle DEF \cong \triangle ABC$ since $\triangle DEF$ can be mapped onto $\triangle ABC$ by a sequence of rigid motions.

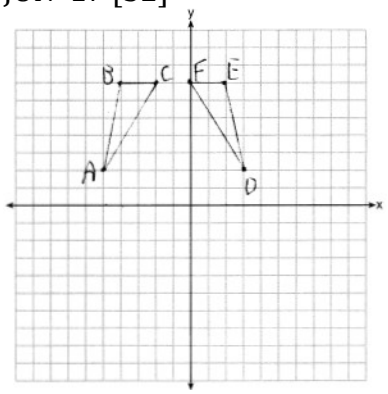
12) JAN '16 [28]
Yes. The sequence of transformations consists of a reflection r_{y-axis} and a translation $T_{0,-3}$, which are rigid motions which preserve congruency.

13) AUG '16 [33]



$(x,y) - \text{rotation point} \rightarrow$
$(-y,x) + \text{rotation point}$
$C(2,-9) - (2,-3) = (0,-6) \rightarrow$
$(6,0) + (2,-3) = C'(8,-3)$
$B(6,-8) - (2,-3) = (4,-5) \rightarrow$
$(5,-4) + (2,-3) = B'(7,1)$
$\triangle DEF \cong \triangle A'B'C'$ because $\triangle DEF$ is a reflection of $\triangle A'B'C'$ over $x = -1$ and reflections preserve congruency.

14) JUN '17 [32]



Reflection $r_{x=-1}$. The triangles are congruent because reflections are rigid motions that preserve distance.

15) AUG '17 [30]
Yes, a sequence of rigid motions preserve distance and angle measure, so $\triangle ABC \cong \triangle XYZ$ by ASA. $\overline{BC} \cong \overline{YZ}$ by CPCTC.

16) JAN '18 [30]
$AB = \sqrt{3^2 + 8^2} = \sqrt{73}$ and $RS = \sqrt{3^2 + 7^2} = \sqrt{58}$, so $\overline{AB}$ is not congruent to $\overline{RS}$. Therefore, $\triangle ABC$ is not congruent to $\triangle RST$. Since they are not congruent, there is no sequence of rigid motions that would map $\triangle ABC$ onto $\triangle RST$.

17) JUN '18 [25]
Yes, translations are rigid motions that preserve distance and angles.

11.4 Transformations and Similarity

Practice Problems

1. $D_{\frac{1}{2}}$ with A as the center of dilation followed by $T_{4,0}$ mapping $A \to A'$.	2. $D_{\frac{1}{2}}$ with B as the center of dilation, then a translation $T_{0,-2}$ to map $B \to B'$, and finally a reflection $r_{\overline{B'C'}}$.

Regents Questions

1) JUN '15 [2] Ans: 4
2) AUG '15 [2] Ans: 3
3) AUG '15 [14] Ans: 4
4) AUG '16 [9] Ans: 4
5) JAN '17 [2] Ans: 2
6) JUN '17 [14] Ans: 1
7) AUG '18 [2] Ans: 1
8) FALL '14 [17]
Circle A can be mapped onto circle B by first translating circle A such that A maps onto B, and then dilating circle A, centered at A, by a scale factor of $\frac{5}{3}$. Since there exists a sequence of transformations that maps circle A onto circle B, circle A is similar to circle B.
9) FALL '14 [19]
Let $\triangle X'Y'Z'$ be the image of $\triangle XYZ$ after a rotation about point Z such that $\overline{ZX'}$ coincides with $\overline{ZU}$. Since rotations preserve angle measure, $\overline{ZY'}$ coincides with $\overline{ZV}$.
Then, dilate $\triangle X'Y'Z'$ by a scale factor of $\frac{ZU}{ZX'}$ with its center at point Z. Since dilations preserve angles, $\overline{X'Y'}$ maps onto $\overline{UV}$.
Therefore, $\triangle XYZ \sim \triangle UVZ$.
10) JUN '16 [34]
A dilation of $\frac{5}{2}$ about the origin.
Dilations preserve similarity.
11) JAN '18 [32]
Dilation by a scale factor of 3 with its center at point A. Dilations preserves similarity.

12. CIRCLES

12.1 Circumference and Rotation

Practice Problems

1. $D = 1000$ ft. $C = 2 \cdot 5\pi = 10\pi$ ft. $R = \frac{D}{C} = \frac{1000}{10\pi} ft \approx 31.8$ It must make at least 32 revolutions.	2. D = 100 ft. = 1200 in. $C = 8\pi$ in. $R = \frac{D}{C} = \frac{1200 in}{8\pi in} \approx 47.7$ 47 clocks can be framed.
3. $C = 2\pi$ $D = C \cdot R = 2\pi \cdot 1100.5 \approx 6{,}914.65\, ft$ $6{,}914.65\, ft \times \frac{1 mile}{5{,}280\, ft} \approx 1.3 miles$	4. D = 2 mi. = 10,560 ft. $C = 394\pi$ ft. $R = \frac{10{,}560\, ft.}{394\pi\, ft.} \approx 8.5$
5. D = 1 mile = 5,280 ft. $C = 2\pi$ $R = \frac{5280}{3 \cdot 2\pi} ft \approx 280.1$ 281 rotations are needed.	

Regents Questions

1) JAN '16 [23] Ans: 1

12.2 Arcs and Chords

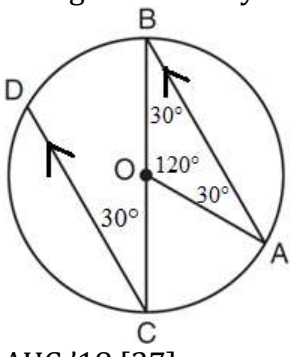
Practice Problems

1. (4) supplementary	2. (4) right
3. $\frac{2}{12}=\frac{1}{6}$ $\quad \frac{1}{6}\cdot 360^\circ = 60^\circ$	4. $\text{m}\angle ABC = 30^\circ$
5. $\text{m}\angle AOC = 48^\circ$	6. $\text{m}\,\widehat{BC} = 140^\circ$ $\text{m}\,\widehat{AC} = 180 - 140 = 40^\circ$
7. parallel chords intercept congruent arcs, so $\text{m}\,\widehat{DB} = \frac{180-110}{2} = 35^\circ$	8. $\frac{180-80}{2} = 50^\circ$
9. $\text{m}\angle A = 90^\circ$, so $\text{m}\angle C = 35^\circ$	10. $\text{m}\widehat{GFE} = 86\times 2 = 172^\circ$ $\text{m}\angle F = 180 - 86 = 94^\circ$
11. $\text{m}\angle ADC = \frac{132+82}{2} = 107^\circ$	12. $15x = 360^\circ$ $x = 24^\circ$ $\text{m}\widehat{FE} = 48^\circ$ and $\text{m}\widehat{GD} = 168^\circ$ $\text{m}\angle D = \text{m}\angle G = \frac{1}{2}\text{m}\widehat{FE} = 24^\circ$ *or* $\text{m}\angle E = \text{m}\angle F = \frac{1}{2}\text{m}\widehat{GD} = 84^\circ$

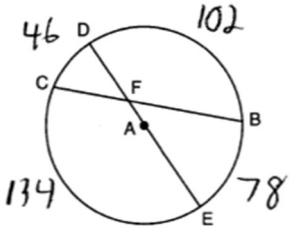
Regents Questions

1) JUN '15 [8] Ans: 1
2) AUG '15 [15] Ans: 3
3) JUN '16 [10] Ans: 2
4) AUG '16 [23] Ans: 1
5) JUN '17 [4] Ans: 4
6) JUN '17 [8] Ans: 2
7) AUG '17 [4] Ans: 1
8) JAN '18 [16] Ans: 4
9) JAN '18 [21] Ans: 4
10) JUN '18 [17] Ans: 3
11) JAN '19 [5] Ans: 4

12) JAN '16 [26]
Ans: 120°
$\angle B \cong \angle C$ (alternate interior angles)
$\angle A \cong \angle B$ (base angles of isosceles triangle formed by two radii)



13) AUG '18 [27]
Ans: 118°

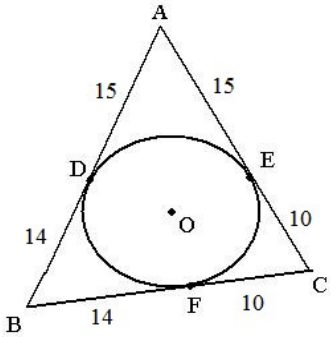


$$\frac{134+102}{2}=118$$

12.3 Tangents

Practice Problems

1. Perimeter = 78.



2. $\triangle ABC$ is a right triangle.
$(AC)^2 = 8^2 + 15^2$
$(AC)^2 = 289$
$AC = 17$

AD is a radius.
$CD = AC - AD = 17 - 8 = 9$

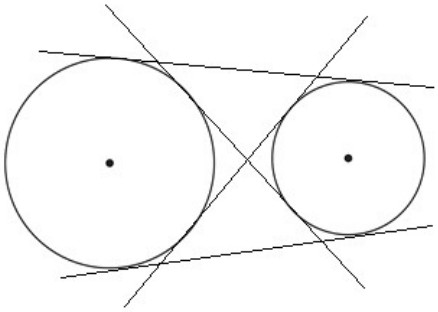
3. $\overline{AB} \cong \overline{AC}$, so $\triangle ABC$ is an isosceles triangle.
$m\angle A = 180 - 2(66) = 48°$

4. $m\overset{\frown}{AB} = 180 - 38 = 142°$,
so $m\angle AOB = 142°$

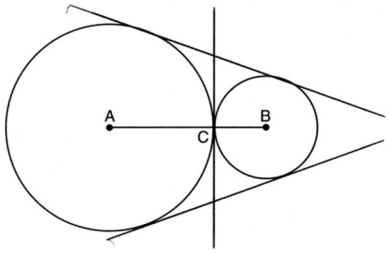
5. $m\overset{\frown}{RS} = 180 - 54 = 126°$.

6. The measure of minor arc DC is
$360 - 246 = 114°$.
$m\angle DEC = 180 - 114 = 66°$.

7.



8.



Regents Questions

1) JUN '15 [20] Ans: 1
2) AUG '15 [12] Ans: 3
3) JAN '16 [21] Ans: 3
4) AUG '18 [14] Ans: 2
5) AUG '16 [25]
$\frac{3}{8} \cdot 56 = 21$

12.4 Secants

Practice Problems

1. (a) tangent (b) secant
(c) diameter (d) chord

2. $x = \dfrac{86-44}{2} = 21°$

3. $\text{m}\angle P = \dfrac{70-20}{2} = 25°$

4. $30 = \dfrac{140 - \text{m}\overset{\frown}{CD}}{2}$, so $\text{m}\overset{\frown}{CD} = 80°$.
$\text{m}\overset{\frown}{DE} = 360 - (140+80) = 140°$.

5.
$(PA)^2 = (PB)(PC)$
$(PA)^2 = (4)(16)$
$(PA)^2 = 64$
$PA = 8$

6.
$(PQ)(PR) = (PS)(PT)$
$(6)(24) = (8)(PT)$
$PT = 18$

5.
$(BC)(AC) = (EC)(DC)$
$(3)(12) = (EC)(9)$
$EC = 4$

6.
$x^2 = 3(x+18)$
$x^2 = 3x + 54$
$x^2 - 3x - 54 = 0$
$(x-9)(x+6) = 0$
$x = 9$

9.
(a) Let $x = TM$.
$x(x+2) = (12)(2)$
$x^2 + 2x - 24 = 0$
$(x+6)(x-4) = 0$
$x = 4$
$RT = 6 + 4 = 10$

(b)
$(PS)^2 = (8)(18)$
$(PS)^2 = 144$
$PS = 12$

10.
(a) $\text{m}\overset{\frown}{ACB} = 360 - (56 + 112) = 192$
$\text{m}\overset{\frown}{CB} = \frac{1}{4} \times 192 = 48$
$\text{m}\angle CEB = \dfrac{56+48}{2} = 52$

(b) $\text{m}\angle F = \dfrac{192-112}{2} = 40$

(c) $\text{m}\angle DAC = \dfrac{112+48}{2} = 80$

Regents Questions

1) JAN '17 [15] Ans: 2

2) AUG '17 [12] Ans: 2

3) JAN '17 [28]

$$\frac{152-56}{2}=48°$$

4) JUN '18 [28]

$$10\cdot 6=15x$$
$$x=4$$

5) JAN '19 [27]

$$\text{m}\angle RPS=\frac{m\widehat{RS}-m\widehat{WT}}{2}$$
$$35=\frac{121-}{2}$$
$$-x=70-121$$
$$x=51°$$

12.5 Circle Proofs

Practice Problems

1.

Statements		Reasons
Circle O, $\overset{\frown}{AB} \cong \overset{\frown}{AC}$		Given
$\overline{AB} \cong \overline{AC}$	(S)	If two arcs are congruent, their chords are congruent
$\overline{AO} \cong \overline{AO}$	(S)	Reflexive Property
$\overline{OC} \cong \overline{OB}$	(S)	All radii in a circle are congruent
$\triangle AOC \cong \triangle AOB$		SSS

Alternately, $\overline{AB} \cong \overline{AC}$ could be replaced with $\angle AOC \cong \angle AOB$ (central angles of congruent arcs are congruent), and then $\triangle AOC \cong \triangle AOB$ by SAS.

2.

Statements		Reasons
Circle Q, $\overline{PQR} \perp \overline{ST}$		Given
$\overline{PQR} \cong \overline{PQR}$	(S)	Reflexive Property
$\angle PRS$, $\angle PRT$ are right angles		$\perp$ segments form right angles
$\angle PRS \cong \angle PRT$	(A)	All right angles are congruent
$\overline{QR}$ bisects $\overline{ST}$		If a radius is perpendicular to a chord, it bisects the chord
$\overline{RS} \cong \overline{RT}$	(S)	Definition of bisector
$\triangle PRS \cong \triangle PRT$		SAS
$\overline{PS} \cong \overline{PT}$		CPCTC

3.

Statements		Reasons
Tangents $\overline{PA}$ and $\overline{PB}$, radii $\overline{OA}$ and $\overline{OB}$, and $\overline{OP}$ intersects the circle at C.		Given
$\overline{OA} \perp \overline{PA}$ and $\overline{OB} \perp \overline{PB}$		a tangent is perpendicular to a radius at the point of tangency
$\angle PAO$ and $\angle PBO$ are right angles		perpendicular lines form right angles
$\overline{OP} \cong \overline{OP}$	(H)	Reflexive property
$\overline{OA} \cong \overline{OB}$	(L)	all radii in a circle are congruent
$\triangle AOP \cong \triangle BOP$		HL
$\angle AOP \cong \angle BOP$		CPCTC

4.

Statements		Reasons
Diameter $\overline{BOD}$, $\text{m}\overset{\frown}{BR} = 70$, $\text{m}\overset{\frown}{YD} = 70$		Given
$\text{m}\angle RDB = \text{m}\angle YBD = 35$	(A)	The measure of an inscribed angle is half its intercepted arc
$\angle BRD$ and $\angle DYB$ are right angles		an inscribed angle of a semicircle is a right angle
$\angle BRD \cong \angle DYB$	(A)	all right angles are congruent
$\overline{BD} \cong \overline{DB}$	(S)	Reflexive property
$\triangle RBD \cong \triangle YDB$		AAS

Alternately, $\overline{BR} \cong \overline{YD}$ (if two arcs are congruent, their chords are congruent), which leads to the triangles congruent by HL

5.

Statements		Reasons
Quadrilateral $ABCD$ inscribed in circle O, $\overline{AB} \parallel \overline{DC}$, diagonals $\overline{AC}$ and $\overline{BD}$		Given
$\overset{\frown}{AD} \cong \overset{\frown}{BC}$		parallel chords intercept congruent arcs
$\angle BDC \cong \angle ACD$	(A)	inscribed angles that intercept congruent arcs are congruent
$\angle DAC \cong \angle DBC$	(A)	inscribed angles that intercept the same arc are congruent
$\overline{AD} \cong \overline{BC}$	(S)	congruent chords intercept congruent arcs
$\triangle ACD \cong \triangle BDC$		AAS

6.

Statements		Reasons
$\overline{AD}$ is a diameter, $\overline{AD} \parallel \overline{BC}$		Given
$\angle BEA \cong \angle CED$	(A)	vertical angles are congruent
$\angle BAC \cong \angle CDB$	(A)	inscribed angles that intercept the same arc are congruent
$\overset{\frown}{AB} \cong \overset{\frown}{DC}$		parallel chords intersect congruent arcs
$\overline{AB} \cong \overline{DC}$	(S)	congruent chords intersect congruent arcs
$\triangle BAE \cong \triangle CDE$		AAS
$\overline{BE} \cong \overline{CE}$		CPCTC

Regents Questions

1) FALL '14 [26]
(Givens omitted)
Chords $\overline{BC}$ and $\overline{BD}$ are drawn (auxiliary lines).
$\angle A \cong \angle A$, $\overset{\frown}{BC} \cong \overset{\frown}{BC}$ (Reflexive property).
$\text{m}\angle BDC = \frac{1}{2}\text{m}\overset{\frown}{BC}$ (an inscribed angle is half the measure of its intercepted arc).
$\text{m}\angle CBA = \frac{1}{2}\text{m}\overset{\frown}{BC}$ (an angle formed by a tangent and chord measures half the intercepted arc).
$\text{m}\angle BDC = \text{m}\angle CBA$ (substitution).
$\angle BDC \cong \angle CBA$ (definition of congruence).
$\triangle ABC \sim \triangle ADB$ (AA~).
$\frac{AB}{AC} = \frac{AD}{AB}$ (Side Proportionality).
$AC \cdot AD = AB^2$ (product of the means equals the product of the extremes).

2) AUG '16 [35]
(Givens omitted)
Chords $\overline{CB}$ and $\overline{AD}$ are drawn (auxiliary lines);
$\angle CEB \cong \angle AED$ (vertical angles);
$\angle C \cong \angle A$ (inscribed angles that intercept the same arc are congruent);
$\triangle BCE \sim \triangle DAE$ (AA~);
$\frac{AE}{CE} = \frac{ED}{EB}$ (Side Proportionality);
$AE \cdot EB = CE \cdot ED$ (The product of the means equals the product of the extremes).

3) AUG '17 [33]
(Givens omitted)
$\angle B$ is a right angle (angle inscribed in semi-circle is a right angle).
$\overleftrightarrow{EC} \perp \overleftrightarrow{OC}$ (radius drawn to a point of tangency is $\perp$ to the tangent).
$\angle ECA$ is a right angle (perpendicular lines form right angles).
$\angle B \cong \angle ECA$ (all right angles are congruent).
$\angle BCA \cong \angle CAE$ (the transversal of parallel lines creates congruent alternate interior angles).
$\triangle ABC \sim \triangle ECA$ (AA~).
$\frac{BC}{CA} = \frac{AB}{EC}$ (Corresponding sides of similar triangles are in proportion).

12.6 Arc Lengths and Sectors

PRACTICE PROBLEMS

1. $\frac{90}{360}=\frac{L}{30\pi}$ $L=7.5\pi$ feet	2. $\frac{40}{360}=\frac{L}{12\pi}$ $L=\frac{4}{3}\pi$ m
3. Central angle = 180 – 150 = 30° $\frac{30}{360}=\frac{L}{34\pi}$ $L\approx 9$ cm	4. $\frac{\theta}{360}=\frac{14\pi}{36\pi}$ $\theta=140°$
5. $\frac{\theta}{360}=\frac{247}{2\pi\cdot 150}$ $\theta\approx 94°$	6. $\frac{165}{360}=\frac{L}{2\pi\cdot 2.4}$ $L\approx 6.9$ meters
7. $\frac{40}{360}=\frac{S}{36\pi}$ $S=4\pi$ sq. units	8. Central angle = 360 – 45 = 315° $\frac{315}{360}=\frac{S}{16\pi}$ $S=14\pi$ sq. cm
9. Area of sector: $\frac{90}{360}=\frac{S}{100\pi}$ $S=25\pi$ Area of triangle: $\frac{1}{2}(10)(10)=50$ Area of segment = $25\pi-50$ sq. units	10. Area of sector: $\frac{120}{360}=\frac{S}{144\pi}$ $S=48\pi$ sq. units Area of triangle: $\frac{1}{2}(12)(12)\sin 120°=36\sqrt{3}$ Area of segment = $48\pi-36\sqrt{3}$ sq. units
11. $\frac{9}{360}=\frac{L}{2\pi\cdot 3954}$ $L\approx 621.1$ miles	

Regents Questions

1) JUN '15 [23] Ans: 2
2) AUG '15 [18] Ans: 3
3) JAN '16 [12] Ans: 3
4) JUN '16 [24] Ans: 3
5) AUG '16 [19] Ans: 2
6) JAN '17 [21] Ans: 4
7) JAN '18 [24] Ans: 3
8) JUN '18 [22] Ans: 4
9) AUG '18 [18] Ans: 2
10) JAN '19 [14] Ans: 2
11) FALL '14 [23]

$$\text{m}\angle BOD = \frac{180-20}{2} = 80°$$

$$\frac{\theta}{360°} = \frac{S}{\pi r^2}$$

$$\frac{80}{360} = \frac{S}{36\pi}$$

$$S = 8\pi$$

12) JUN '15 [29]

$$\frac{\theta}{360°} = \frac{S}{\pi r^2}$$

$$\frac{\theta}{360} = \frac{12\pi}{36\pi}$$

$$\theta = 120°$$

13) JUN '17 [26]

$$\frac{\theta}{360°} = \frac{S}{\pi r^2}$$

$$\frac{40}{360} = \frac{S}{20.25\pi}$$

$$S = 2.25\pi$$

14) JAN '18 [28]

$$S = 625\pi - 500\pi = 125\pi$$

$$\frac{\theta}{360°} = \frac{S}{\pi r^2}$$

$$\frac{\theta}{360°} = \frac{125\pi}{625\pi}$$

$$\theta = \tfrac{1}{5} \cdot 360° = 72°$$

12.7 Radians (–)

Practice Problems

1. (a) $45\cdot\frac{\pi}{180}=\frac{\pi}{4}$ rad (b) $270\cdot\frac{\pi}{180}=\frac{3\pi}{2}$ rad (c) $150\cdot\frac{\pi}{180}=\frac{5\pi}{6}$ rad	2. (a) $\frac{\pi}{6}\cdot\frac{180}{\pi}=30°$ (d) $\frac{5\pi}{9}\cdot\frac{180}{\pi}=100°$ (b) $\frac{5\pi}{4}\cdot\frac{180}{\pi}=225°$ (e) $\frac{8\pi}{5}\cdot\frac{180}{\pi}=288°$ (c) $\frac{3\pi}{5}\cdot\frac{180}{\pi}=108°$
3. $2.5\left(\frac{180}{\pi}\right)\approx 143.2°$	4. $216\left(\frac{\pi}{180}\right)\approx 3.8$ rad
5. $65=5r$ r = 13 feet	6. $20=2.5r$ r = 8 ft.
7. $4\pi=\frac{2\pi}{3}r$ r = 6 ft.	8. $18=\frac{3}{4}r$ r = 24 cm
9. $L = 2\cdot 4 = 8$ inches	10. $L=\frac{\pi}{4}\cdot 12=3\pi\approx 9.4$ inches
11. $L=\frac{4\pi}{3}\cdot 12=16\pi\approx 50$ cm	12. $L=\frac{2\pi}{5}\cdot 18=7.2\pi\approx 23$ inches
13. $8\pi=\theta\cdot 10$ $\theta=\frac{4\pi}{5}$ rad	14. $\frac{\theta}{2\pi}=\frac{S}{\pi r^2}$ $\frac{0.97}{2\pi}=\frac{S}{81\pi}$ S = 39.285 cm^2

Regents Questions

1) FALL '14 [4] Ans: 3

2) AUG '17 [23] Ans: 2

3) JUN '16 [29]

$\pi = A\cdot 4$

$A=\frac{\pi}{4}$

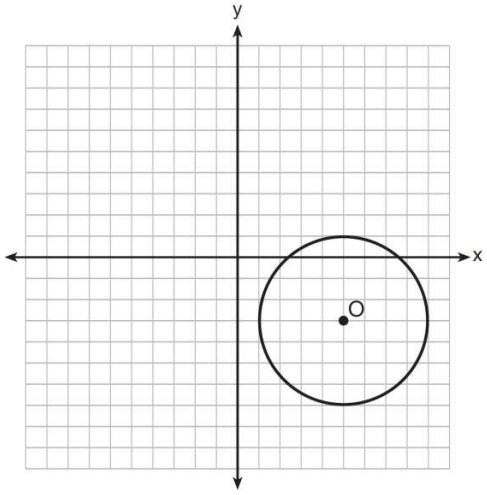
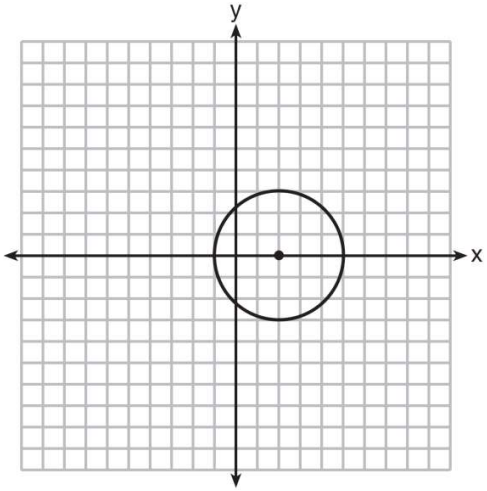
$\frac{13\pi}{8}=B\cdot 6.5$

$B=\frac{\pi}{4}$

Yes, both angles are equal.

12.8 Circles in the Coordinate Plane

Practice Problems

1. (a) $(0,0)$ and $\sqrt{10}$ (b) $3^2+(-1)^2=10$ Yes.	2. $(2,-3)$ and 6
3. $(1,-3)$ and 3	4. $(0,7)$ and $4\sqrt{2}$
5. $r=4$ $x^2+y^2=16$	6. $r^2=3^2+4^2$ $r=5$ $(x+3)^2+(y-4)^2=25$
7. 	8. 
9. $r=3$ $(x-1)^2+(y+2)^2=9$	10. Center is at the midpoint of the diameter, $(2,-4)$. Radius is 4. $(x-2)^2+(y+4)^2=16$
11. $(3,-2)\rightarrow(9,-6)$	12. $(-4,2)\rightarrow(-8,4)$ and $r=2\cdot 9=18$
13. $x^2+4x+y^2=5$ $(x^2+4x+4)+y^2=5+4$ $(x+2)^2+y^2=9$ Center is $(-2,0)$, radius is 3	14. $x^2+6x+y^2-4y=12$ $(x^2+6x+9)+y^2-4y=12+9$ $(x+3)^2+y^2-4y=21$ $(x+3)^2+(y^2-4y+4)=21+4$ $(x+3)^2+(y-2)^2=25$ Center is $(-3,2)$, radius is 5

15. $x^2 - 16x + y^2 + 6y = -53$ $(x^2 - 16x + 64) + y^2 + 6y = -53 + 64$ $(x-8)^2 + y^2 + 6y = 11$ $(x-8)^2 + (y^2 + 6y + 9) = 11 + 9$ $(x-8)^2 + (y+3)^2 = 20$ Center is (8,–3), radius is $2\sqrt{5}$	16. $x^2 - 2x = -y^2 + 10y + 1$ $x^2 - 2x + y^2 - 10y = 1$ $(x^2 - 2x + 1) + y^2 - 10y = 1 + 1$ $(x-1)^2 + y^2 - 10y = 2$ $(x-1)^2 + (y^2 - 10y + 25) = 2 + 25$ $(x-1)^2 + (y-5) = 27$ Center is (1,5), radius is $3\sqrt{3}$

REGENTS QUESTIONS

1) JUN '15 [14] Ans: 2
2) AUG '15 [9] Ans: 3
3) JAN '16 [17] Ans: 4
4) JUN '16 [3] Ans: 2
5) JUN '16 [23] Ans: 1
6) AUG '16 [16] Ans: 1
7) JAN '17 [18] Ans: 1
8) JAN '17 [22] Ans: 3
9) JUN '17 [12] Ans: 1
10) JAN '18 [12] Ans: 2
11) JUN '18 [20] Ans: 2
12) AUG '18 [21] Ans: 4
13) JAN '19 [20] Ans: 1

14) AUG '16 [30]
Yes.

$$(x-1)^2 + (y+2)^2 = 4^2$$
$$(3.4-1)^2 + (1.2+2)^2 = 4^2$$
$$5.76 + 10.24 = 16 \checkmark$$

15) AUG '17 [31]

$$x^2 - 6x + y^2 + 8y = 56$$
$$x^2 - 6x + 9 + y^2 + 8y + 16 = 56 + 9 + 16$$
$$(x-3)^2 + (y+4)^2 = 81$$

(3,–4); $r = 9$

13. SOLIDS

13.1 Volume

Practice Problems

1. $V = Bh = 6 \cdot 8 \cdot 4 = 192$ ft^3	2. $V = e^3 = (1.5)^3 = 3.375$ cm^3
3. $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(9)^3 = 972\pi$ m^3	4. $V_{before} = \frac{4}{3}\pi(3)^3 = 36\pi$ $V_{after} = \frac{4}{3}\pi(6)^3 = 288\pi$ $288\pi - 36\pi = 252\pi$ in^3
5. $V = \pi r^2 h = \pi(4)^2(10) \approx 502.65$ in^3	6. $V = \pi r^2 h = \pi(6)^2(15) \approx 1696.5$ in^3
7. $V = \frac{1}{3}Bh = \frac{1}{3}(10)(8)(6) = 160$ in^3	8. $V = \frac{1}{3}(12)^2(42) = 2{,}016$ m^3
9. $V = \pi r^2 h$ $32\pi = \pi r^2(2)$ $r^2 = 16$ $\quad r = \sqrt{16} = 4$ in.	10. $V = \frac{1}{3}Bh$ $256 = \frac{1}{3}B(12)$ $B = 64$ $\quad s = \sqrt{64} = 8$
11. $V = 3 \cdot 8 = 24$ cm^3	12. $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)^2(8) = 24\pi$ in^3
13. $(x-2)(x+1)(2x)$ $= (x^2 - x - 2)(2x)$ $= 2x^3 - 2x^2 - 4x$	14. $V = 5^3 + \frac{1}{3}(25)(6) = 175$ cm^3
15. $V = 5^3 = 125$ $\quad V = 10^3 = 1000$ $1000 \div 125 = 8$	16. $V = \pi r^2 h$ $342 = 9\pi h$ $\quad h = \frac{38}{\pi}$ $36 \div \frac{38}{\pi} = \frac{36\pi}{38} \approx 2.97$ $\quad$ 2 cans
17. $V_{larger} = Bh = 12 \cdot 30 \cdot 16 = 5760$ $\quad V_{smaller} = Bh = 6 \cdot 12 \cdot 9 = 648$ $5760 - 648 = 5{,}112$ in^3	
18. $V_{container} = (20)(15)(10) = 3{,}000$ in^3 $\quad V_{cup} = \pi(2)^2(5) = 20\pi$ in^3 $3{,}000 \div 20\pi \approx 47.7$ $\quad$ 47 cups	
19. $V_{prism} = Bh$ $\quad V = (5)(3.5)(7) = 122.5$ $V_{cylinder} = \pi r^2 h$ $\quad V = \pi(2.5)^2(7) \approx 137.4$ Cylinder by 14.9 in^3	

Regents Questions

1) AUG '15 [21] Ans: 4
2) JAN '16 [4] Ans: 2
3) JAN '16 [7] Ans: 2
4) JAN '16 [14] Ans: 3
5) JUN '16 [6] Ans: 4
6) AUG '16 [20] Ans: 4
7) JAN '17 [11] Ans: 2
8) JAN '17 [24] Ans: 1
9) JUN '17 [16] Ans: 1
10) JUN '17 [18] Ans: 3
11) JAN '18 [7] Ans: 3
12) JAN '18 [22] Ans: 2
13) JUN '18 [7] Ans: 1
14) JUN '18 [10] Ans: 1
15) AUG '18 [19] Ans: 2
16) JAN '19 [9] Ans: 2
17) JAN '19 [23] Ans: 1
18) JUN '16 [32]

$$\frac{\pi(11.25)^2(33.5)}{231} \approx 57.7$$

19) JUN '16 [36]

Similar triangles are required to model and solve a proportion.

Let x = height of the rest of the cone

$$\frac{height\ of\ cone}{radius\ of\ top} = \frac{height\ below\ glass}{radius\ of\ bottom}$$

$$\frac{x+5}{1.5} = \frac{x}{1} \qquad x = 10$$

$h = 10 + 5 = 15$

$$V_{glass} = V_{cone} - V_{below\ glass}$$

$$V = \tfrac{1}{3}\pi(1.5)^2(15) - \tfrac{1}{3}\pi(1)^2(10)$$

$$\approx 24.9$$

20) JAN '17 [34]

$$C = 2\pi r$$

$$31.416 = 2\pi r$$

$$r = \frac{31.416}{2\pi} \approx 5$$

$$V = \tfrac{1}{3}\pi r^2 h = \tfrac{1}{3}(25)(13)\pi \approx 340$$

21) JUN '17 [28]

$V = \frac{4}{3}\pi r^3$, so $r = \sqrt[3]{\frac{3V}{4\pi}}$.

$$\sqrt[3]{\frac{3(294)}{4\pi}} - \sqrt[3]{\frac{3(180)}{4\pi}} \approx 0.6$$

22) JUN '17 [34]

$$V = 20{,}000g \cdot \frac{1\ \text{ft}^3}{7.48\ g} \approx 2673.8\ \text{ft}^3$$

$$V = \pi r^2 h$$

$$2673.8 = \pi r^2(34.5)$$

$$r \approx 4.967$$

$$p \approx 2(4.967) + 1$$

$$p \approx 10.9$$

23) JAN '18 [33]

$$V_{hemisphere} = \tfrac{1}{2} \cdot \tfrac{4}{3}\pi r^3$$

$$= \tfrac{2}{3}\pi \cdot 4^3 \approx 134.04$$

$$V_{cylinder} = \pi r^2 h$$

$$= \pi \cdot 4^2 \cdot (13-4) \approx 452.39$$

$$V \approx 134.04 + 452.39 \approx 586\,\text{m}^3$$

24) JUN '18 [31]

$$2\pi r = 29.5$$

$$r = \frac{29.5}{2\pi}$$

$$V = \tfrac{4}{3}\pi r^3 = \tfrac{4}{3}\pi\left(\frac{29.5}{2\pi}\right)^3 \approx 434\ \text{in}^3$$

25) AUG '18 [31]

$$2\left(\frac{36}{12} \times \frac{36}{12} \times \frac{4}{12}\right) \times \$3.25$$

$$= \$19.50$$

26) JAN '19 [33]

$V_T = 30 \cdot 15 \cdot 3.5 = 1{,}575$ cu. ft.

$1575 \cdot 7.48 = 11{,}781$ gallons

$\$3.95 \cdot 117.81 \approx \$465.35$

$V_N = \pi \cdot 12^2 \cdot 3.5 = 504\pi$ cu. ft.

$504\pi \cdot 7.48 \approx 11{,}843.553$ gallons

\$200 per 6000 = \$1 per 30 gallons

$11{,}843.553 \div 30 \approx \$394.79$

Theresa paid more.

13.2 Density

Practice Problems

1.

$$W = VD$$
$$= 100\ \text{cm}^3 \cdot \frac{11.34\ \text{g}}{1\ \text{cm}^3}$$
$$= 1{,}134\ \text{g}$$

2.

$$D = \frac{W}{V} = \frac{89.6\ \text{g}}{10\ \text{cm}^3} = 8.96\ \text{g/cm}^3$$

3.

$$D = \frac{W}{V} = \frac{94.44\ \text{g}}{12\ \text{cm}^3} = 7.87\ \text{g/cm}^3$$

4.

$$V = \frac{W}{D} = \frac{9.8}{0.7} = 14\ \text{mL}$$

5.

$$V = \frac{1}{3}Bh = \frac{1}{3}(24)(10) = 80\ \text{in}^3$$

$$W = 80\ \cancel{\text{in}^3} \cdot \frac{2\ \cancel{\text{g}}}{1\ \cancel{\text{cm}^3}} \cdot \frac{(2.54)^3\ \cancel{\text{cm}^3}}{1\ \cancel{\text{in}^3}} \cdot \frac{1\ \cancel{\text{kg}}}{1000\ \cancel{\text{g}}} \cdot \frac{2.2\ \text{lbs}}{1\ \cancel{\text{kg}}}$$

$$\approx 5.8\ \text{lbs}$$

Regents Questions

1) JUN '15 [7] Ans: 3
2) AUG '15 [16] Ans: 1
3) JAN '16 [19] Ans: 2
4) JUN '16 [18] Ans: 2
5) JUN '16 [20] Ans: 1
6) AUG '16 [17] Ans: 2
7) FALL '14 [6]

$5.1\times10.2\times20.3=1{,}056.006\ \text{cm}^3$

$500\times1{,}056.006=528{,}003\ \text{cm}^3$

$$528{,}003\text{cm}^3\times\frac{1\text{m}^3}{100\text{cm}^3}=0.528003\text{m}^3$$

$$\frac{1920\text{kg}}{\text{m}^3}\times0.528003\text{m}^3\approx1013\text{kg}$$

No, the weight of the bricks is greater than 900 kg

8) FALL '14 [25]

$$r=25\text{cm}\times\frac{1\text{m}}{100\text{cm}}=0.25\text{m}$$

$V=\pi(0.25)^2(10)=0.625\pi\ \text{m}^3$

$$W=0.625\pi\,\text{m}^3\times\frac{380\text{kg}}{\text{m}^3}\approx746.1\text{kg}$$

$\$4.75\times746.1\approx\$3{,}544$ per tree

$$\frac{\$50{,}000}{\$3{,}544}\approx14.1$$

Need to sell 15 trees

9) JUN '15 [35]

$$\tan47°=\frac{x}{8.5}\qquad x\approx9.115$$

$V_{cone}=\frac{1}{3}\pi(8.5)^2(9.115)\approx689.6$

$V_{cylinder}=\pi(8.5)^2(25)\approx5674.5$

$V_{hemisphere}=\frac{1}{2}\left(\frac{4}{3}\pi(8.5)^3\right)\approx1286.3$

$V_{tower}\approx689.6+5674.5+1286.3$

$\approx7650\text{ft}^3$

$$85\%\times7650\text{ft}^3\times\frac{62.4\text{lb}}{\text{ft}^3}=405{,}756\text{lb}$$

No, the weight exceeds 400,000 lbs

10) AUG '15 [25]

$\frac{137.8}{6^3}\approx0.638\text{g/cm}^3$; Ash

11) AUG '15 [36]

$V_{cone}=\frac{1}{3}\pi\left(\frac{3}{2}\right)^2(8)\approx18.85\text{in}^3$

$\text{Total}\approx100\times18.85\approx1885\text{in}^3$

$(0.52\times1885)\times\$0.10=\$98.02$

$(100\times\$1.95)-(\$98.02+\$37.83)$

$=\$59.15$

12) JAN '16 [30]

Dish A

$$d_A=\frac{40{,}000}{\pi(25.5)^2}\approx19.6$$

$$d_B=\frac{72{,}000}{\pi(37.5)^2}\approx16.3$$

13) AUG '16 [36]

$V=\frac{1}{3}\pi\left(\frac{8.3}{2}\right)^2(10.2)+\frac{1}{2}\cdot\frac{4}{3}\pi\left(\frac{8.3}{2}\right)^3$

$\approx183.961+149.693\approx333.65\ \text{cm}^3$

$333.65\times50=16{,}682.7\text{cm}^3$

$16{,}682.7\times0.697=11{,}627.8\text{g}$

$11{,}627.8\,g=11.6278\,kg$

$11.6278\times3.83=\$44.53$

14) JAN '17 [36]

$V_{cylinder}$

$=\pi(26.7)^2(750)-\pi(24.2)^2(750)$

$=95{,}437.5\pi$

$$95{,}437.5cm^3\times\frac{2.7\,g}{1cm^3}\times\frac{1\,kg}{1000\,g}\times\frac{\$0.38}{1\,kg}$$

$=\$307.62$

V_{prism}

$=(40^2)(750)-(35^2)(750)=281{,}250$

$$281{,}250cm^3\times\frac{2.7\,g}{1cm^3}\times\frac{1\,kg}{1000\,g}\times\frac{\$0.38}{1\,kg}$$

$=\$288.56$

Prisms cost less.

$307.62-288.56=\$19.06$

15) AUG '17 [36]

$$\tan 16.5 = \frac{x}{13.5}$$
$$x \approx 4$$
$$4.5 + 4 = 8.5\,\text{ft}$$
$$V_1 = 9 \times 16 \times 4.5 = 648$$
$$V_2 = 13.5 \times 16 \times 4.5 = 972$$
$$V_3 = \tfrac{1}{2} \times 13.5 \times 16 \times 4 = 432$$
$$V_4 = 12.5 \times 16 \times 8.5 = 1700$$
$$V_1 + V_2 + V_3 + V_4 = 3752\ \text{ft}^3$$
$$3752 - (35 \times 16 \times 0.5) = 3472$$
$$3472 \times 7.48 \approx 25{,}971$$
$$\frac{25{,}971}{10.5} \approx 2473.4$$
$$\frac{2473.4}{60} \approx 41\,\text{hrs}$$

16) JAN '18 [29]

$D = \frac{W}{V}$, so $7.95 = \frac{W}{1015}$

$W = (7.95)(1015) = 8{,}069.25$ g

$500 \times 8069.25 = 4{,}034{,}625\ \text{g} = 4034.625\ \text{kg}$

$4034.625 \times 0.29 \approx \$1{,}170$

17) JUN '18 [34]

$$V = \pi r^2 h = \pi(10^2)(18) = 1800\pi$$
$$1800\pi\,\text{in}^3 \cdot \frac{1\,\text{ft}^3}{12^3\,\text{in}^3} \approx 1.0417\pi$$
$$1.0417\pi(95.46)(0.85) \approx 266\ \text{lbs}$$

266 + 270 = 536 lbs

18) AUG '18 [34]

Diameter of hollow is $4 - 2(0.5) = 3$.

Radius of outer sphere is 2 and radius of inner sphere is 1.5.

$$V = \frac{4}{3}\pi(2^3 - 1.5^3) \approx 19.4$$
$$19.4 \times 1.308 \times 8 \approx 203$$

13.3 Lateral Area and Surface Area

Practice Problems

1.
$SA = 2lw + 2hl + 2hw$
$= 2 \cdot 8 \cdot 6 + 2 \cdot 4 \cdot 8 + 2 \cdot 4 \cdot 6$
$= 96 + 64 + 48 = 208$ ft^2

2.
$SA = 2lw + 2hl + 2hw$
$= 2(3)(1.5) + 2(2)(3) + 2(2)(1.5)$
$= 9 + 12 + 6 = 27$ ft^2

3.
$SA = 2lw + 2hl + 2hw$
$= 2(3.0)(2.2) + 2(7.5)(3.0) + 2(7.5)(2.2)$
$= 13.2 + 45 + 33 = 91.2$ cm^2

4.
$SA = 2lw + 2hl + 2hw$
$= 2(5.5)(3) + 2(5.5)(6.75) + 2(3)(6.75)$
$= 33 + 74.25 + 40.5 = 147.75$ cm^2

5. $V = 10 \cdot 2 \cdot 4 = 80\,\text{cm}^3$
$SA = 2 \cdot 10 \cdot 2 + 2 \cdot 10 \cdot 4 + 2 \cdot 2 \cdot 4$
$= 136\,\text{cm}^2$

6. $s = \sqrt[3]{64} = 4$
$SA = 6 \cdot 4^2 = 96$ in^2

7.
$LA = 2\pi rh$ $\quad$ $SA = 2\pi r^2 + LA$
$= 2\pi \cdot 6 \cdot 9$ $\quad$ $= 72\pi + 108\pi$
$= 108\pi$ ft^2 $\quad$ $= 180\pi$ ft^2

8.
$LA = 2\pi rh$ $\quad$ $SA = 2\pi r^2 + LA$
$= 2\pi \cdot 5 \cdot 11$ $\quad$ $= 50\pi + 110\pi$
$= 110\pi\,\text{ft}^2$ $\quad$ $= 160\pi \approx 502.7\,\text{ft}^2$

9.
$2(x+3)(x-4)$ $\quad$ $2x^2 - 2x - 24$
$+2(x+3)(5)$ $\quad$ $10x + 30$
$+2(x-4)(5)$ $\quad$ $+ \;\; 10x - 40$
$2x^2 + 18x - 34$

10. $V = \pi r^2 h$, so $h = \dfrac{V}{\pi r^2}$

Radius r	Height h	Surface Area SA
2	$h = \dfrac{V}{\pi r^2} = \dfrac{144\pi}{4\pi} = 36$	$SA = 2\pi r^2 + 2\pi rh$ $= 8\pi + 144\pi = 152\pi$
4	$h = \dfrac{V}{\pi r^2} = \dfrac{144\pi}{16\pi} = 9$	$SA = 2\pi r^2 + 2\pi rh$ $= 32\pi + 72\pi = 104\pi$
6	$h = \dfrac{V}{\pi r^2} = \dfrac{144\pi}{36\pi} = 4$	$SA = 2\pi r^2 + 2\pi rh$ $= 72\pi + 48\pi = 120\pi$

The cylinder with the radius of 4 has the least surface area.

Regents Questions

1) JUN '15 [19] Ans: 2

13.4 Rotations of Two-Dimensional Objects

Practice Problems

1. sphere

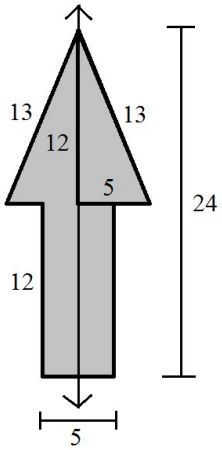
2. cone

3. cylinder
diameter = 10 in and height = 5 in
$V = \pi r^2 h = \pi(5)^2(5) = 125\pi$ in^3

4. cylinder with a hemisphere on top
$V_{cylinder} + V_{hemisphere}$
$= \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$
$= 27\pi + 18\pi = 45\pi$

5. 12 inch radius sphere with a 6 inch radius spherical cutout in its center
$V_{outer\,sphere} - V_{cutout}$
$= \frac{4}{3}\pi(12)^3 - \frac{4}{3}\pi(6)^3$
$= 2{,}304\pi - 288\pi = 2{,}016\pi$

6. cylinder with a cone on top



Cone has a height of 24 – 12 = 12. Use the Pythagorean Theorem to find the radius of the cone:
$r^2 + 12^2 = 13^2$
$r = 5$

Cylinder has radius of 2.5 and height of 12.

$V_{cone} = \frac{1}{3}\pi(5)^2(12) = 100\pi$

$V_{cylinder} = \pi(2.5)^2(12) = 75\pi$

$V_{object} = V_{cone} + V_{cylinder} = 175\pi$.

Regents Questions

1)	JUN '15 [1]	Ans: 4	6)	AUG '17 [13]	Ans: 3
2)	AUG '15 [3]	Ans: 4	7)	JAN '18 [10]	Ans: 4
3)	JUN '16 [1]	Ans: 3	8)	JUN '18 [16]	Ans: 3
4)	AUG '16 [3]	Ans: 1	9)	AUG '18 [3]	Ans: 4
5)	JUN '17 [18]	Ans: 1	10)	JAN '19 [11]	Ans: 3

13.5 Cross Sections

Practice Problems

1. (4)	2. (2)
3. (1)	4. (4)
5. (1)	6. (a) pentagon (b) rectangle

Regents Questions

1) JUN '15 [6] Ans: 2
1) JAN '16 [1] Ans: 1
2) AUG '16 [13] Ans: 3
3) JAN '17 [23] Ans: 4
4) AUG '17 [1] Ans: 2
5) JAN '18 [5] Ans: 2
6) AUG '18 [5] Ans: 3

13.6 Cavalieri's Principle (–)

Practice Problems

1. $V = Bh = (4)(3)(6) = 72 \text{ in}^3$	2. $V = \pi r^2 h = \pi(2)^2(5) = 20\pi \text{ in}^3$

Regents Questions

1) FALL '14 [18]
Each quarter in both stacks has the same base area. Therefore, each corresponding cross-section of the stacks will have the same area. Since the two stacks of quarters have the same height of 23 quarters, the two volumes must be the same.

2) JUN '17 [27]
Each triangular prism has the same base area. Therefore, each corresponding cross-section of the prisms will have the same area. Since the two prisms have the same height of 14, the two volumes must be the same.

3) AUG '17 [25]
Yes. The bases of the cylinders have the same area and the cylinders have the same height.

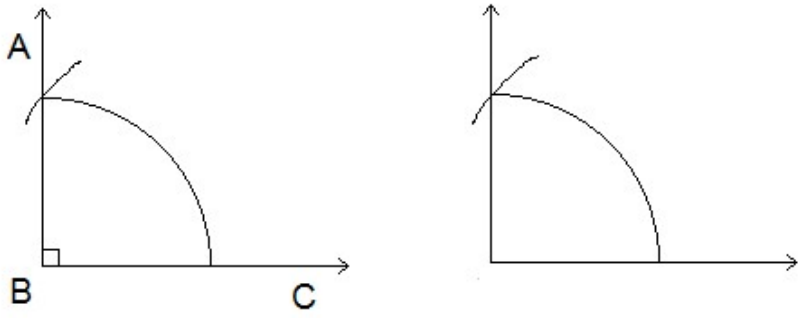
14. CONSTRUCTIONS

14.1 Copy Segments, Angles, and Triangles

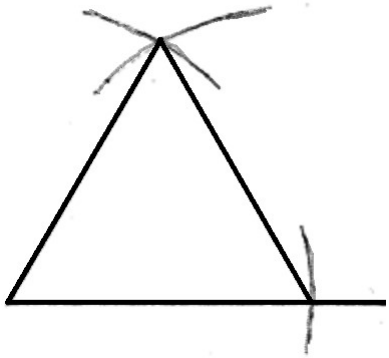
Practice Problems

1.

2.



3.

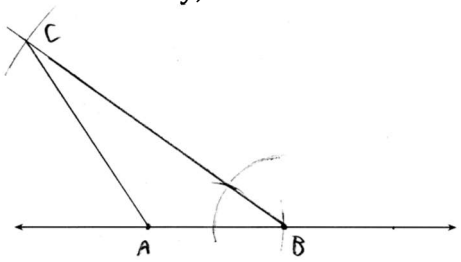


4. SAS

5. AAS

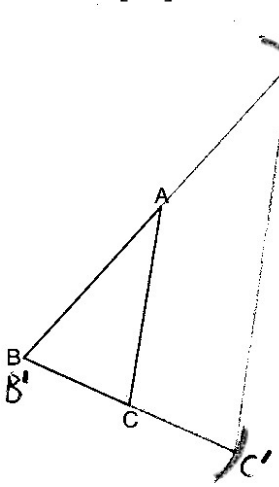
Regents Questions

1) JAN '16 [34]
solutions vary, such as



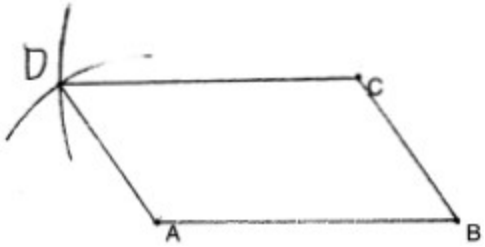
SAS

2) AUG '16 [32]



$A'C'$ is twice AC

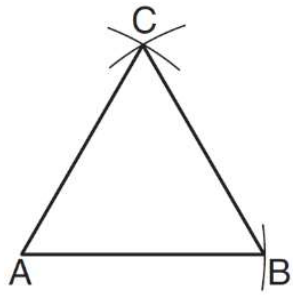
3) JAN '19 [29]
copy length of $\overline{AB}$ from C and length of $\overline{BC}$ from A for point D at intersecting arcs



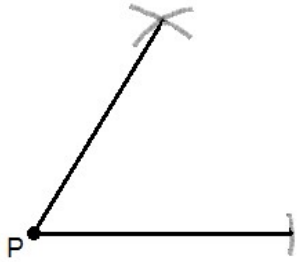
14.2 Construct an Equilateral Triangle

Practice Problems

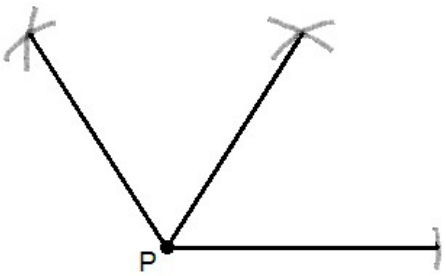
1.



2.



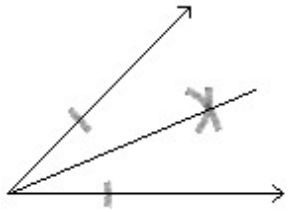
3.



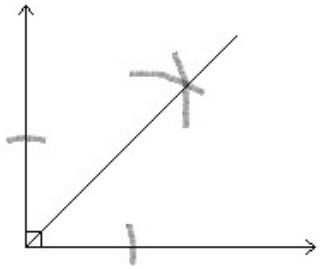
14.3 Construct an Angle Bisector

Practice Problems

1.



2.



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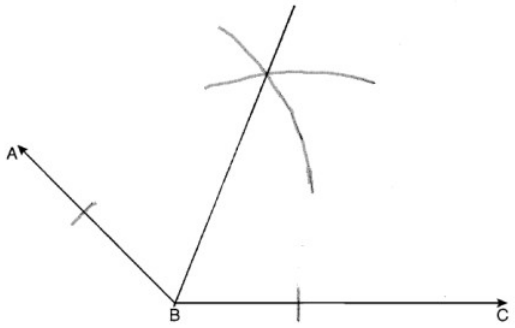
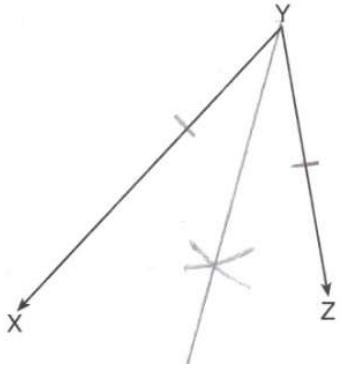


4.



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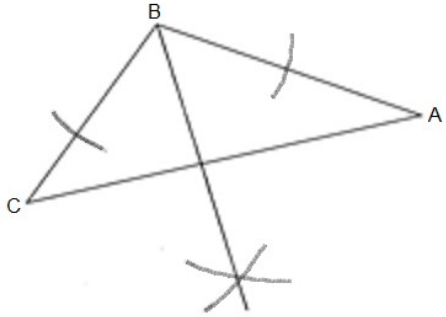

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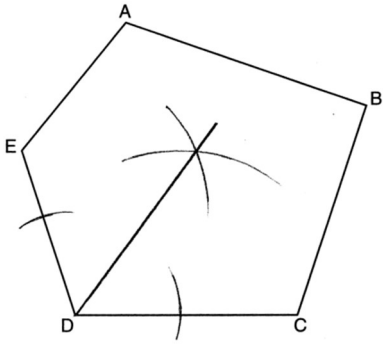

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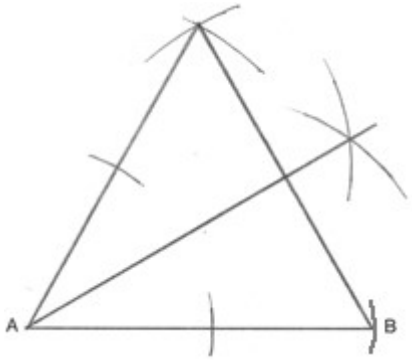
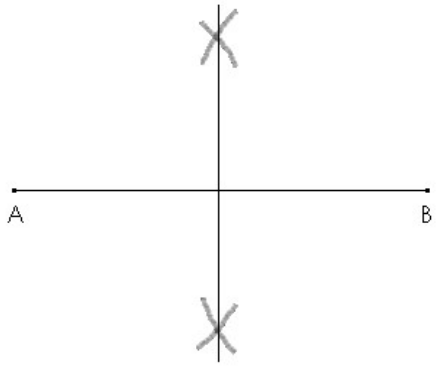


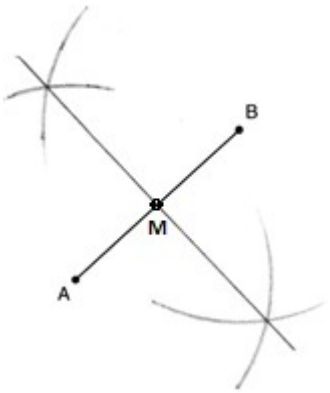
14.4 Construct a Perpendicular Bisector

Practice Problems

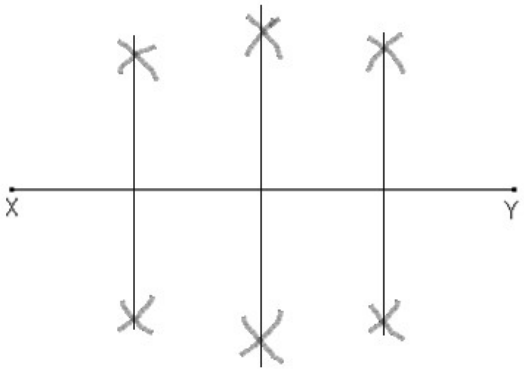
1.



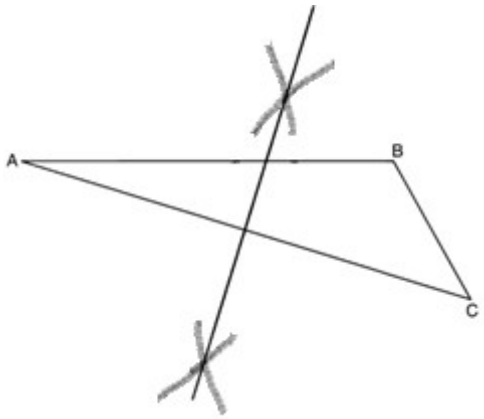
2.



3. Construct the perpendicular bisector of $\overline{XY}$. Then, construct the bisector of each half.

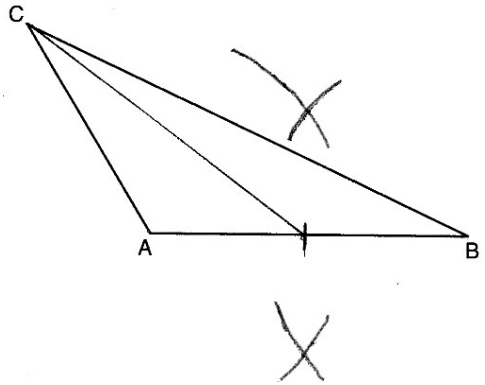


4.

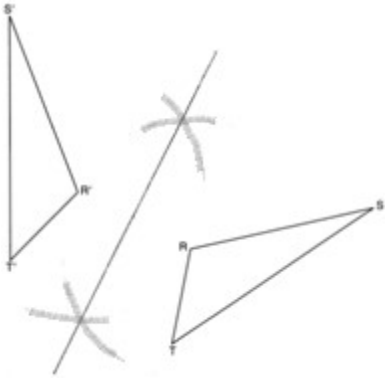


Regents Questions

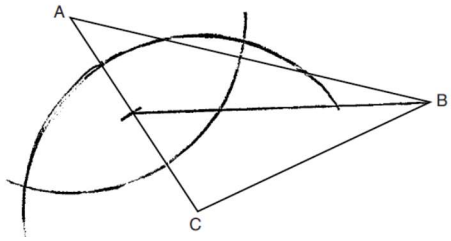
1) AUG '16 [28]



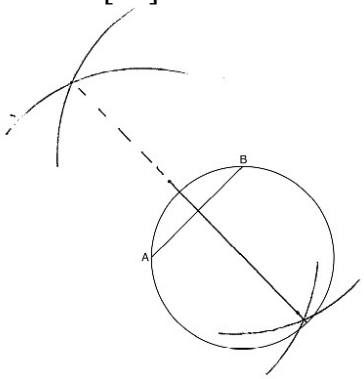
2) JAN '17 [25]



3) JUN '18 [29]



4) AUG '18 [25]

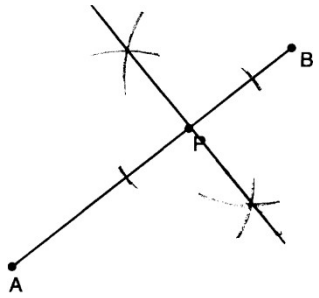


14.5 Construct Lines Through a Point

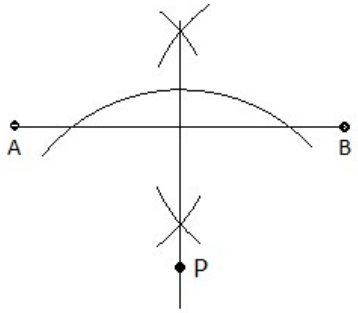
Practice Problems

1. (2)

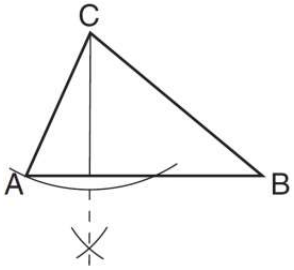
2.



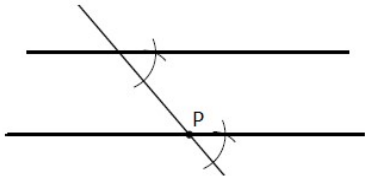
3.



4.



5.

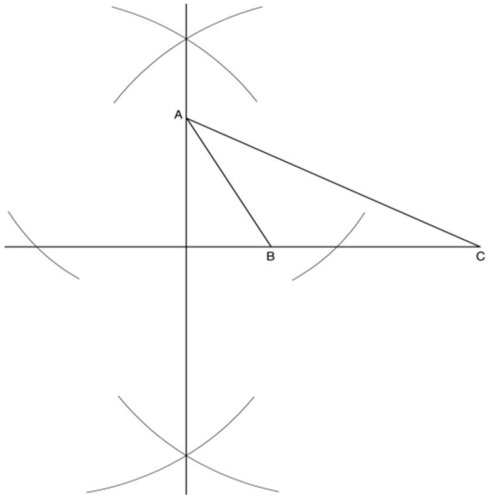


6.

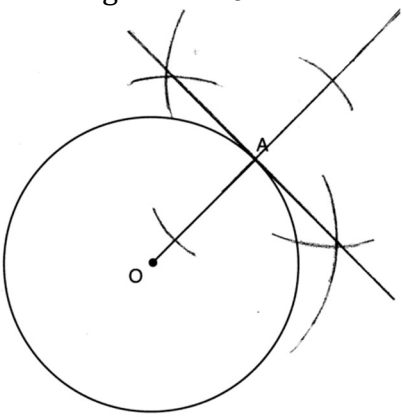
(a) 5 (b) 2 (c) 1 (d) 4 (e) 3 (f) 6

Regents Questions

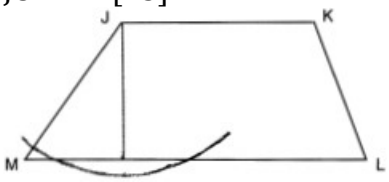
1) FALL '14 [9]
The altitude is perpendicular to the extended side.



2) JUN '16 [31]
The tangent is $\perp \overline{OA}$.



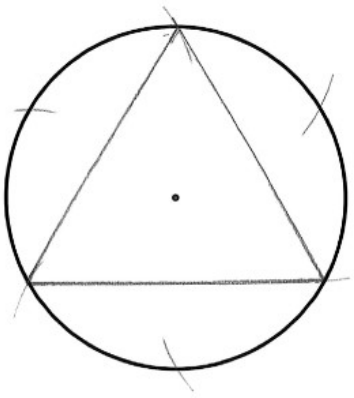
3) JUN '17 [25]



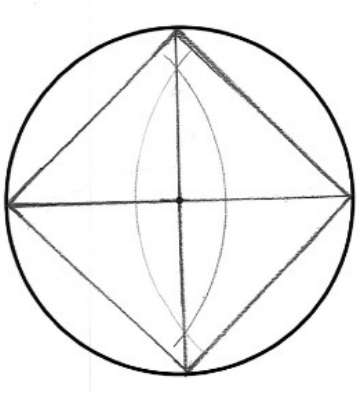
14.6 Construct Inscribed Regular Polygons

Practice Problems

1.

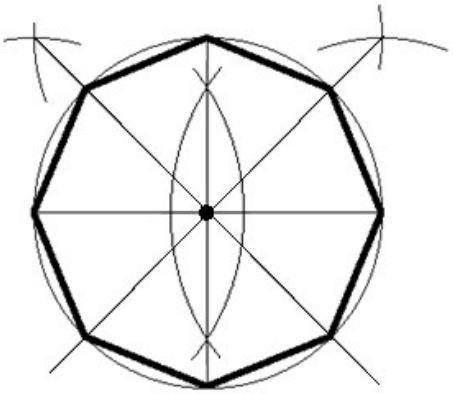


2.



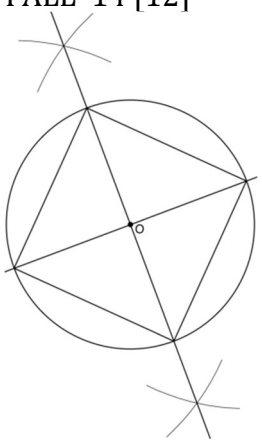
3.

a) Construct two perpendicular diameters (as if constructing an inscribed square).
b) Construct angle bisectors of the 90° central angles, forming eight 45° central angles.
c) The endpoints of the four diameters are the vertices of the regular octagon. Draw the chords between them.



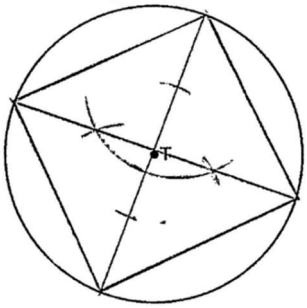
Regents Questions

1) FALL '14 [12]

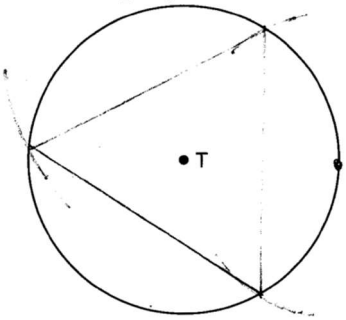


The sides of the square are four congruent chords in the circle, so they intercept four congruent arcs. Each arc therefore measures one-fourth of 360°, or 90°. Therefore, an arc intercepted by two adjacent sides measures $2 \times 90° = 180°$.

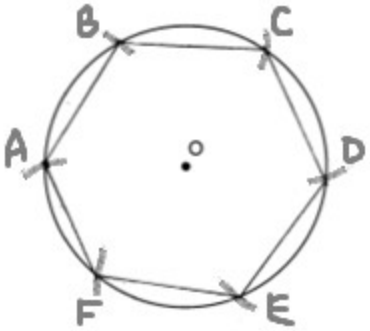
2) JUN '15 [25]



3) AUG '15 [26]

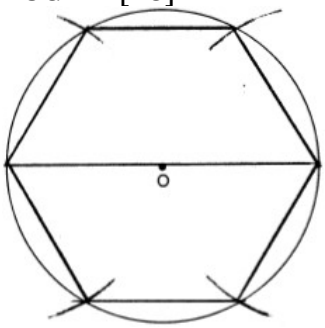


4) JAN '17 [33]

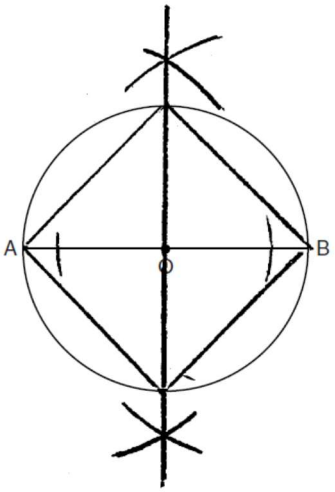


$\overline{COF}$ is a diameter, so $\angle FBC$ is an inscribed angle of a semicircle, and is therefore a right angle. This means $\triangle FBC$ is a right triangle.

5) AUG '17 [28]



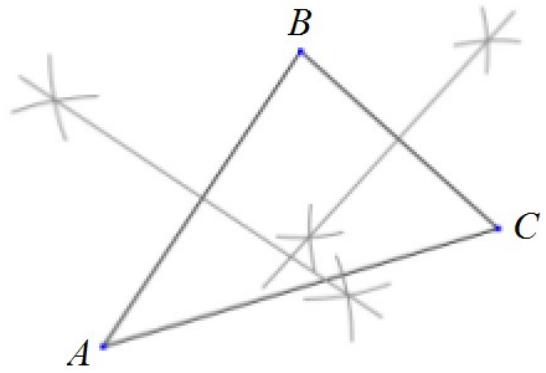
6) JAN '18 [26]



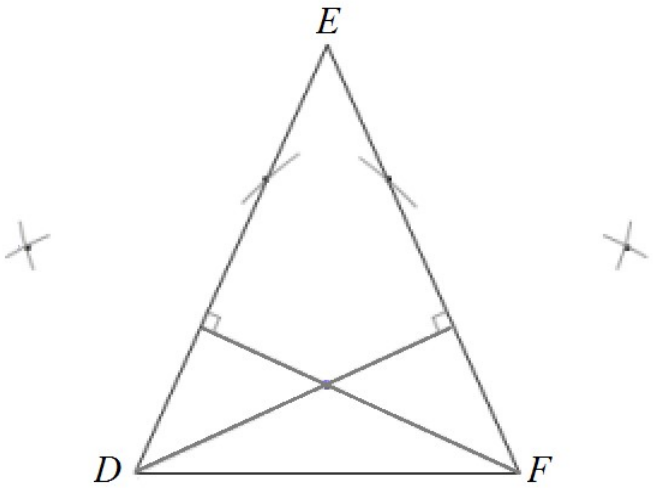
14.7 Construct Points of Concurrency

Practice Problems

1.



2.



14.8 Construct Circles of Triangles

Practice Problems

1. (1)	2. (4)
3. (a) 2 (b) 1 (c) 5 (d) 4 (e) 3	